Identification of mass location on vibrating beams using Haar wavelets and neural networks

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Summary In this paper two procedures are suggested for estimating location and/or magnitude of the concentrated mass attached to the isotropic vibrating beam. Artificial neural networks are applied to establish the mapping relationship between structural feature vector and status of the concentrated mass (location and magnitude). Seven different training methods are applied and compared.

Introduction

Several techniques for estimating the elastic parameters of beams, plates and shells have been proposed by a number of authors [1-3]. These methods include wave propagation measurements, eigenfrequency-based methods, genetic algorithms, etc. In recent years, the artificial neural networks have become a powerful tool in the fields of forecast because of the special abilities to make mappings and simulations of complicate systems and functions. Artificial neural networks have been applied for solving the inverse prediction problems with non-linearity. The supervised multi-layer feed-forward neural network is one of the most popular architecture used today [4]. It is a universal approximator and is taken as the benchmark for comparing the performance of other neural network architectures. The structural parameter estimation based on neural networks includes the following steps: selection of network parameters, determination of network structure, collection and normalisation of learning samples, initialisation of network weight values, to perform the training in order to obtain the convenient accuracy.
In the present work, the dynamic response of vibrating beams with an attached mass is studied. Karlik and Ozkaya [5] applied neural networks to predict five natural frequencies of a beam if the mass ratios and locations were known. The basic idea in the vibration-based estimation is that these parameters depend on the physical properties of the systems structure. Changes in the mass ratio and location can result in detectable alterations in the natural frequencies, displacements or mode shapes. The key problem is how to extract useful features from the vibration signals for identification. However, successful network learning and its ability to generalise characteristic features of the system from input-output pairs requires large training sets.
The aim of the present research is to elaborate two methods which are capable of calculating: (i) the mass ratio if position of the applied mass is known, (ii) the position of the attached mass if mass ratio is known, (iii) the ratio and position of the attached mass.
In the case of the first method the input vector of network consists of five natural frequencies of the system. The network is trained with different training algorithms. A comparison of the algorithms is done in order to find the most accurate method for solving the stated problem.
The second method uses a combined approach: the structural feature vector is calculated with the aid of Haar wavelets. Wavelet transform has been used in many fields including vibration-based damage detection of beams and plates [2, 6-8]. The wavelet packets and neural network identification were suggested by Hein [9] to inversely determine the elastic foundation parameters of delaminated vibrating beams. In most cases the continuous wavelet transform has been used. Non-sufficient attention has been paid to the discontinuous Haar functions, which are

mathematically the simplest wavelets. Nevertheless it has been demonstrated that the Haar wavelets can be successfully applied for solving differential and integral equations [10, 11].
In the present study the integrated method of Haar wavelets and neural networks is suggested. The main ideas of Chen-Hsiao method [10] are applied, according to which the derivatives of the functions were approximated for solving differential equations. This approach has been developed further by Lepik [11].

Dynamic response of vibrating beams carrying the concentrated mass

In this section, an analytical solution to the free vibration of a beam with a concentrated mass located at $x=a$ is formulated. The geometry of the beam is shown in Fig. 1.

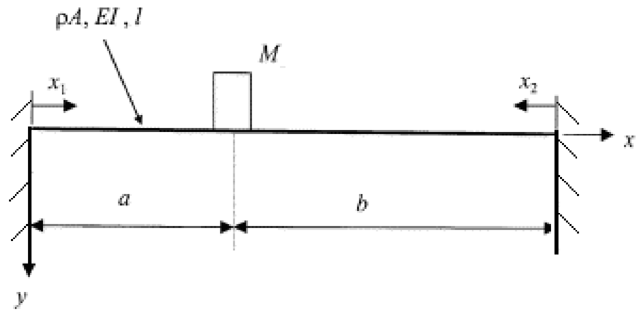

Fig. 1: The beam-mass system.

The differential equation associated with the present eigenvalue problem is [12]:

$$\frac{d^4V}{dx^4} - k^4V = 0, \quad k^4 = \frac{\rho A\omega^2}{EI}. \tag{1}$$

In (1) ρ denotes the beam's density, A is the cross-sectional area, E is Young's modulus and I is the moment of inertia.
The general solutions of the ordinary differential equation (1) can be presented as

$$\begin{cases} V_1(x_1) = C_1\sin(kx_1) + C_2\cos(kx_1) + C_3\sinh(kx_1) + C_4\cosh(kx_1), & x_1 \in [0,a] \\ V_2(x_2) = C_5\sin(kx_2) + C_6\cos(kx_2) + C_7\sinh(kx_2) + C_8\cosh(kx_2), & x_2 \in [0,b] \end{cases} \tag{2}$$

where V_1 and V_2 are the left and right transverse displacements with respect to the concentrated mass M, and C_i (i = 1, …, 8) are the constants to be determined from boundary and continuity conditions. In the present work the following boundary conditions are considered: (i) $V = V' = 0$ (clamped end); (ii) $V'' = V''' = 0$ (free end); (iii) $V' = V''' = 0$ (guided end). Here primes denote differentiation with respect to the spatial variable x. The compatibility conditions at the location of concentrated mass, which apply to all cases [13, 14], are given as follows:

$$\begin{aligned} &V_1(a) = V_2(b), \\ &V_1''(a) = V_2''(b), \\ &V_1'(a) = -V_2'(b), \\ &V_1'''(a) + V_2'''(b) + \alpha k^4 V_1(a) = 0. \end{aligned} \tag{3}$$

where α is the mass ratio defined by $M/(\rho Al)$.

Identification using artificial neural networks

For identification problem the feed-forward back propagation network is used. The neural network contains only one hidden layer whose neurons are assigned the log-sigmoid transfer function. The network is trained by seven different training algorithms in order to find out the most efficient one. In order to compare the performances of the network, trained by different methods, several criteria are used. These are the number of epochs, the mean squared error (MSE), training time, the mean absolute error ($MAE = (1/N)\Sigma|n_t - n_c|$), the variance account for ($VAF = 1 - \mathrm{var}(n_t - n_c)/\mathrm{var}(n_t)$) and the coefficient of multiple determination ($R^2 = 1 - \Sigma(n_t - n_c)^2/ \Sigma(n_t - n_m)^2$). Here n_t is the target output value, n_c is the computed value, n_m is the mean of the target values n_t , N is the number of patterns in the test set; *var* denotes the variance.

Performance assessment of the neural network models trained by natural frequencies

The accuracy of seven methods to predict the attached mass location on a beam with clamped ends is shown in Table 1. The most reliable forecasts have been made by the network which was trained by the Levenberg-Marquardt method. The network made 99.98 percent reliable predictions within 3.7 seconds. The least accurate methods were the steepest gradient methods. Among the conjugate gradient methods the most efficient was the Polak-Ribiére, whose variance account for was almost the same as the Levenberg-Marquardt's.

Table 1. Prediction of mass location on the beam with clamped ends; $\alpha = 10$.

	Sequential mode	Batch mode	Resilient method	Polak-Ribiére	Fletcher-Powell	Powell-Beale	Levenberg Marquardt
No of epochs	NA	1800	127	133	628	172	7
MSE	NA	0.0086	0.9903	0.9.954	0.9.991	0.0003	2.2e-5
Training time	1.0150	16.469	2.3590	2.7810	8.9070	3.3900	3.7030
MAE	0.0268	0.0102	0.0013	0.0012	0.0013	0.0018	0.0008
VAF	0.4425	0.9636	0.9991	0.9997	0.9991	0.9990	0.9999
R^2	0.2244	0.9559	0.9991	0.9994	0.9989	0.9987	0.9998

Performance assessment of the neural network models trained by Haar coefficients

First, the response of the vibrating beam was calculated numerically. Various possible combinations of beam parameters were considered. Second, the vibration responses (mode shapes) of the beam with and without concentrated mass were expanded into Haar series [10, 11]. A comparison of energy of vibration responses between beams with and without mass in some frequency bands will exhibit some remarkable difference. The input vectors for artificial neural networks were calculated with the aid of energy values of sub-signals. The data training sets for artificial neural networks were formed from the input vectors and of corresponding mass locations and ratios.

Table 2. Predictions of the attached body location on a beam; $\alpha = 20$.

	Sequential mode	Batch mode	Resilient method	Polak-Ribiére	Fletcher-Powell	Powell-Beale	Levenberg-Marquardt
No of epochs	NA	1800	36	102	87	64	2
MSE	NA	0.0044	9.03e-5	9.97e-5	9.92e-5	9.74e-5	1.95e-6
Training time	0.7810	20.688	1.3440	1.8440	2.4380	1.6720	0.8910
MAE	0.1074	0.0115	0.0017	0.0008	0.0010	0.0005	0.0002
VAF	1.1348	0.9666	0.9995	0.9998	0.9998	1.0000	1.0000
R^2	0.8308	0.9976	0.9999	1.0000	1.0000	1.0000	1.0000

The results of the predictions made by the networks, trained by seven different methods, are presented in Table 2. During the learning nine-element patters were used. As a result, the

Levenberg-Marquardt and the steepest gradient methods showed the best results. Finally, the tests showed that the training time and number of epochs in second approach were smaller than in the case of previous method.

Concluding remarks

In this work two methods to inversely estimate the attached mass ratio and location on the vibrating beams were proposed. Numerical simulations showed that the integrated approach with Haar wavelets saved the computation time and showed the better accuracy.

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