A BRIEF INTRODUCTION

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OUTLINE

- **General conceptions**
- **Two historical cases**
- **Chaos in different fields**
- **Conclusion**

CHAOS – (Greece *khaos*)

Anaxagoras (~500 – 428 BC) – chaos is the set of main elements from what the universe was created

Platon (427 – 347 BC) – chaos is darkness

Aristoteles (384 – 322 BC) – chaos is space that captures everything

Deterministic mechanics

Isaac Newton (1643 - 1727)

• **Every motion can be described by a set of the differerential equations;**

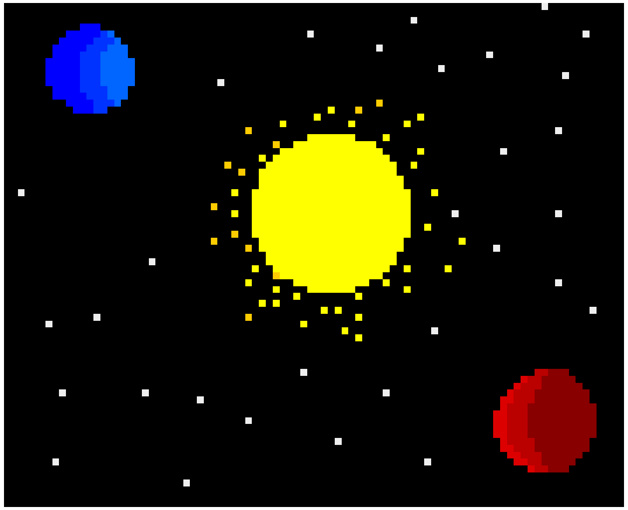
• **If we know the equations, we can calculate the state of the system at every moment (in the future or in the past).**

Systems

A system is something of interest that we are trying to describe or predict

Examples of systems:

- **The Solar system;**



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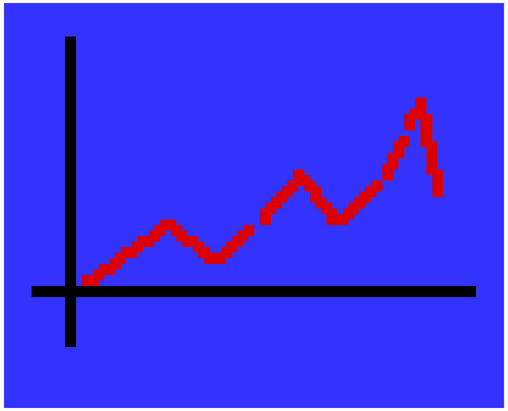
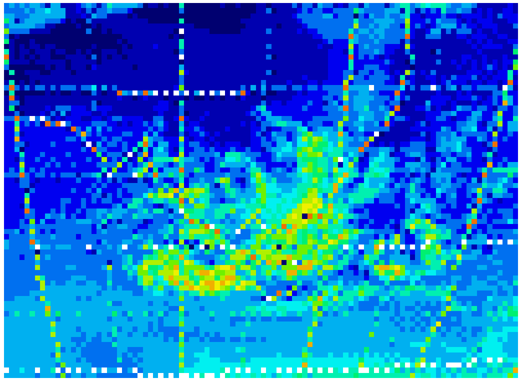
- **The Stock market;**



- **The weather in Estonia;**



- **An ecosystem, e.g. Soomaa National Park;**

CASE I

Edward Lorenz (1917- 2008) US

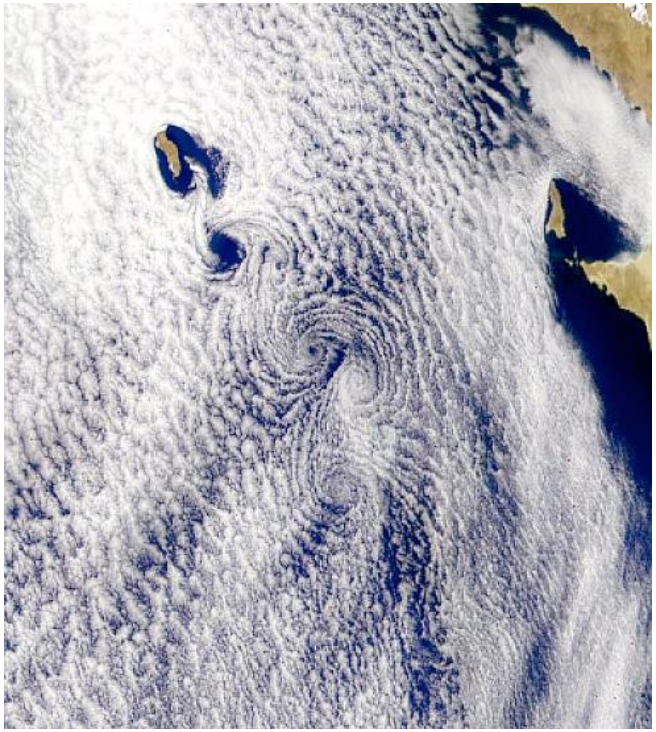
- **Graduated Darthmouth College**
- **In 1960 worked as meteorologist**
- **The problem was whether prediction**
- **He had a computer with a set of twelve equations to model the weather**

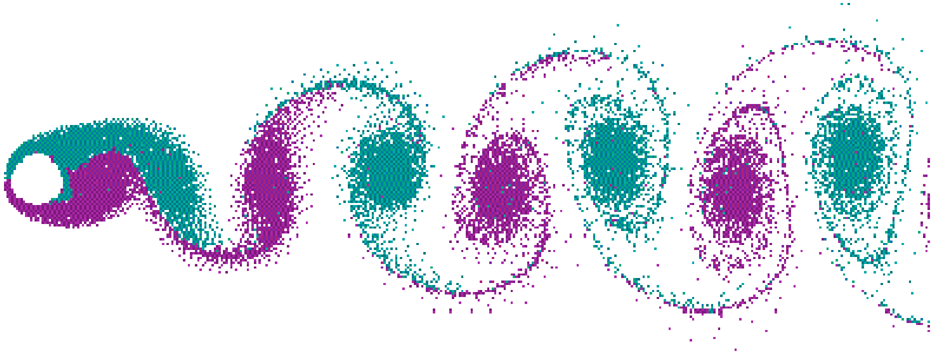
- **He simplified the system up to three equations:**

$$\frac{dx}{dt} = \sigma y - \sigma x$$

$$\frac{dy}{dt} = rx - y - xz$$

$$\frac{dz}{dt} = xy - bz$$





- **He solved the system with computer**

- **He had taken σ = 10; r =1; b = 8/3; y(0) = 0.506127**

- **He wanted to repeat the computations, but for convenience y(0) = 0.506**

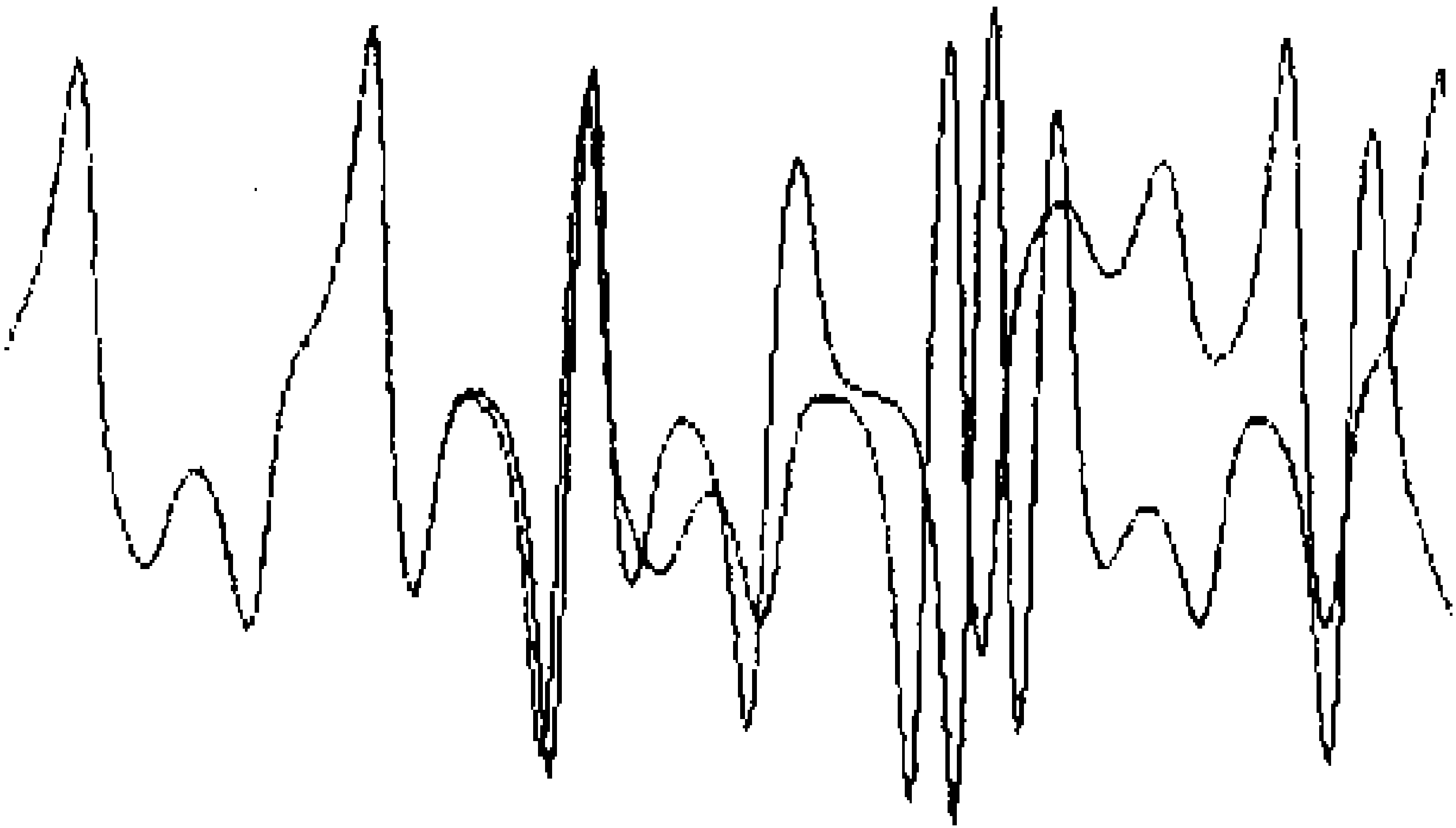


Figure 1: Lorenz's experiment: the difference between the start of these curves is only .000127. (Ian Stewart, Does God Play Dice? The Mathematics of Chaos, pg. 141)

- **He thought: computer is out of order**

- **He repeated calculations again and again …**

- **But the result was the same**

- **He presented the trajectory in space coordinates:**

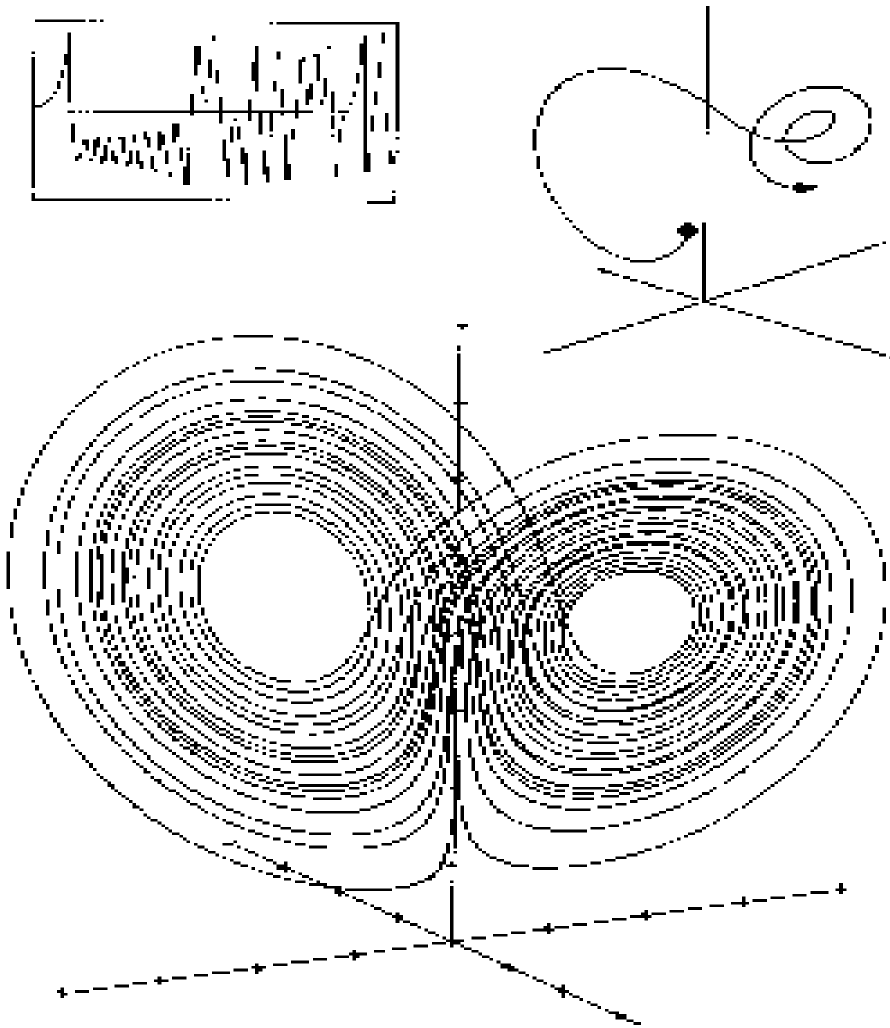
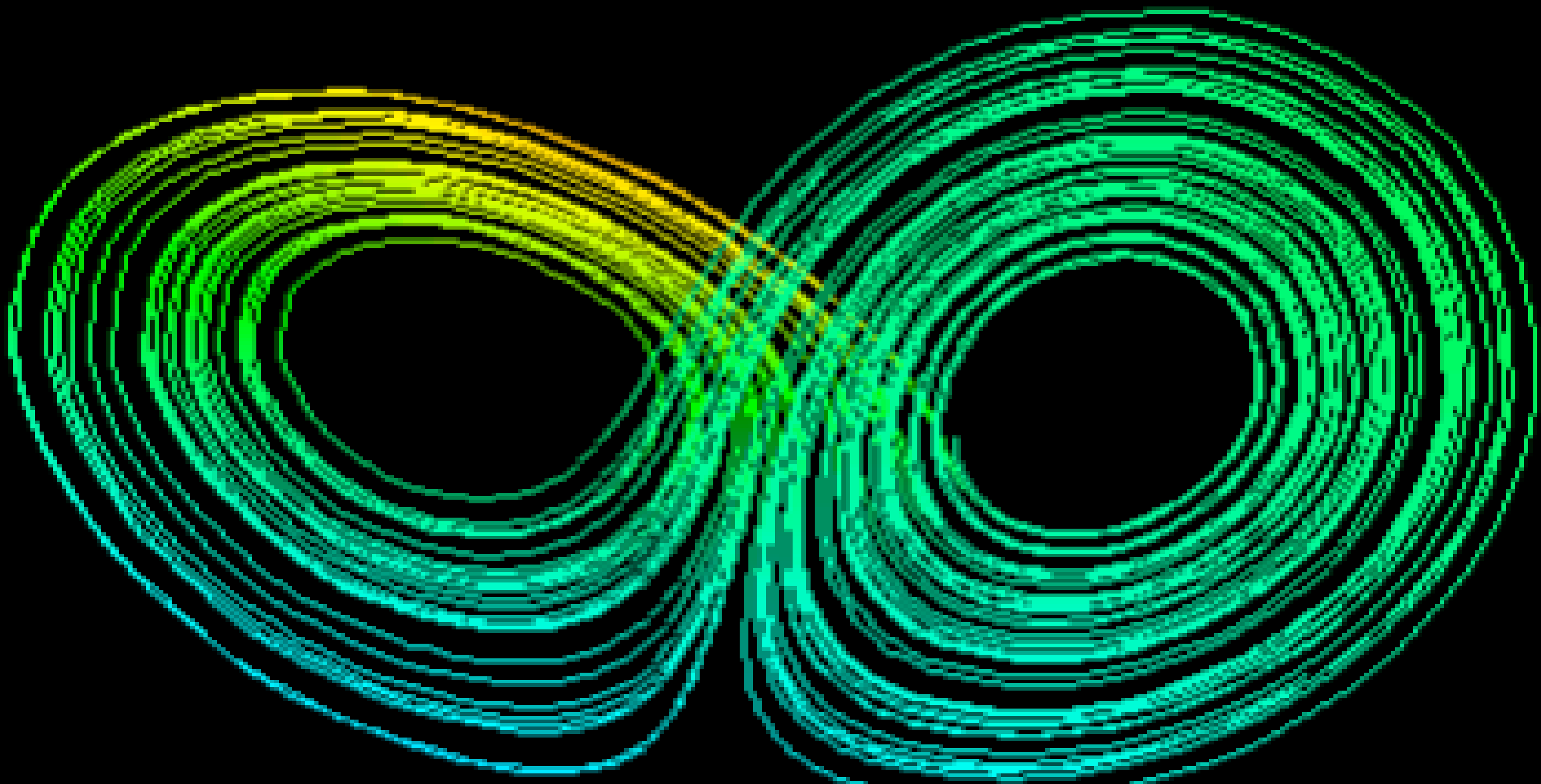


Figure 2: the Lorenz Attractor (James Gleick, Chaos - Making a New Science, pg.29)



Butterfly effect

Predictability: Can the flapping of a single butterfly's wing in Brasil produce tornado in Texas?

Sensitive dependence on initial conditions

**“For want of a nail, the shoe was lost;
For want of a shoe, the horse was lost;
For want of a horse, the rider was lost;
For want of a rider, the battle was lost;
For want of a battle, the kingdom was lost!”**

- A poem in folklore.

CASE II

Yoshisuke Ueda, PhD student, Tokyo

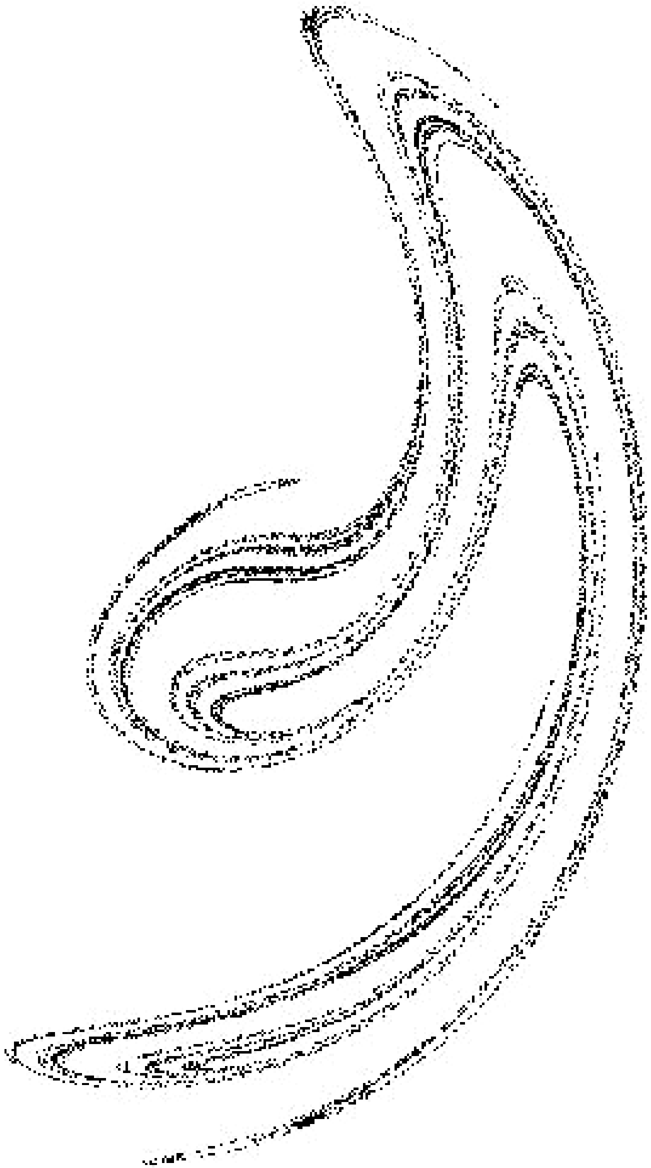
In Nov, 1961 he considered the Duffing system:

$$\frac{d^2x}{dt^2} + h\frac{dx}{dt} + \Omega_0^2 x + \mu x^3 = F\cos\omega t$$

- **He studied the case $\Omega_0 = 0$, $\mu = \omega = 1$ and tried to find subperiods in solution.**

- **His program plotted points of solution at moments 0, T, 2T, … ($T = 2\pi/\omega$)**

- **He got the unexpected picture:**

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- **It was very strange, nonperiodic, irregular, random-like oscillation**

- **Later it was named “strange attractor”**

- **his first paper on this topic was published at 1979**

- **now Duffing eqation in case $\Omega_0 = 0$ is called the Ueda equation**

CHAOS THEORY

The name "chaos theory" comes from the fact that the systems that the theory describes are apparently disordered, but chaos theory is really about finding the underlying order in apparently random data.

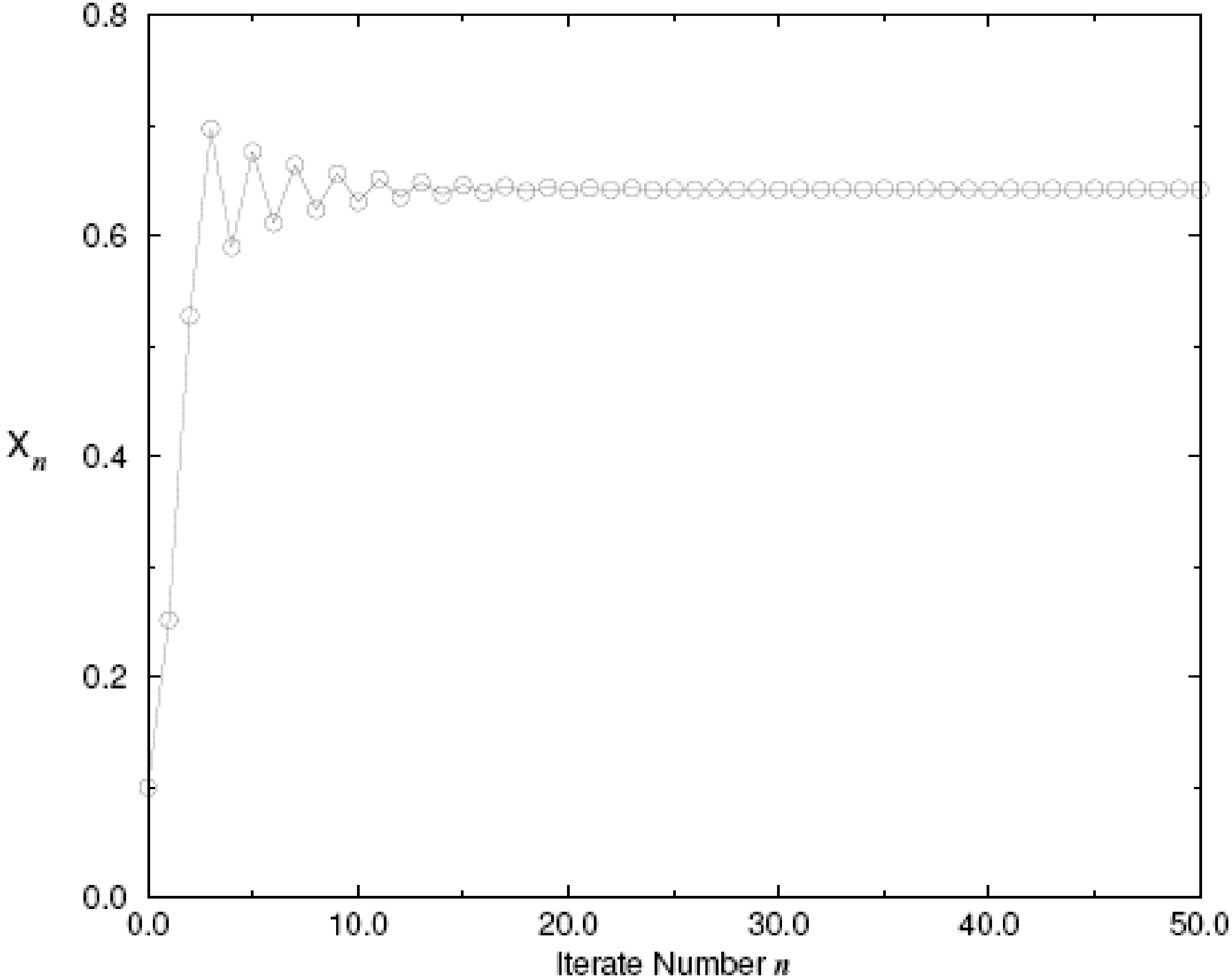
Logistic map
(P.F. Verhulst 1845)

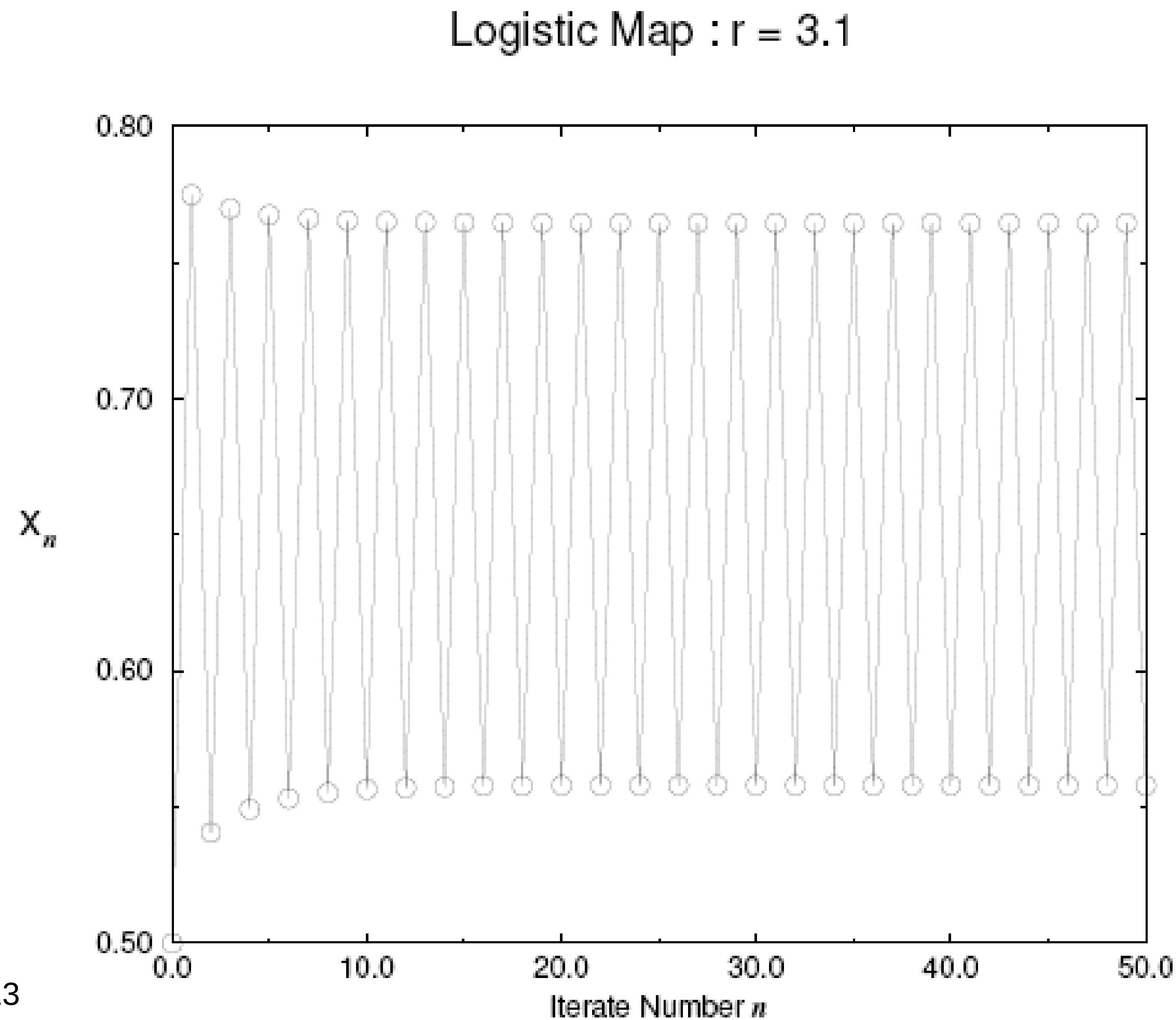
$$x_{n+1} = r\, x_n(1 - x_n)$$

next year's population = *r* * this year's population * (1 - this year's population)

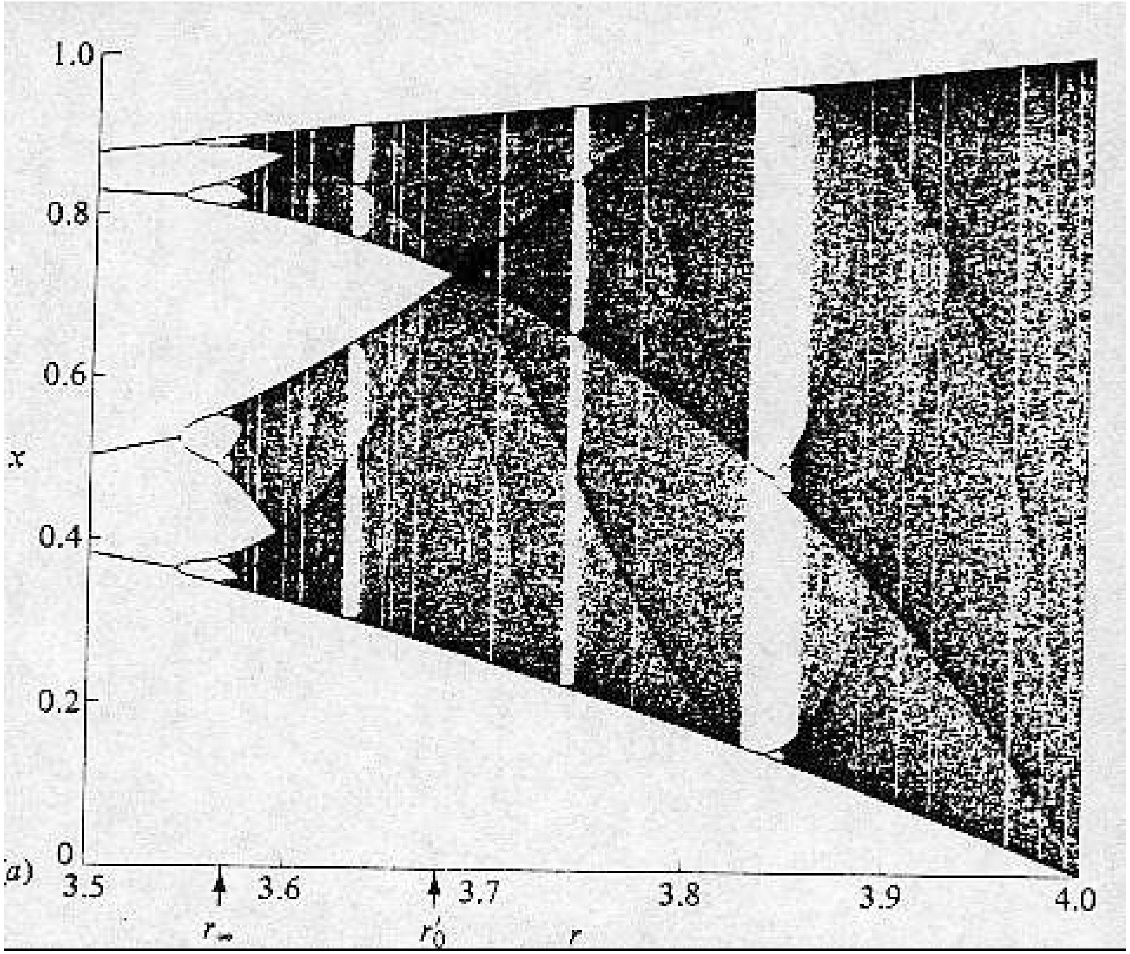
The graph:

Logistic Map : r = 2.8





Logistic Map : r = 3.1
0.80
0.70
0.60
0.50
0.0
10.0
20.0
30.0
40.0
50.0
Iterate Number n
Xn



The bifurcation diagram for the logistic map

Feigenbaum numbers

Mitchell Feigenbaum (1944, Philadelphia)

$$\delta_i = \frac{r_i - r_{i+1}}{r_{i+1} - r_{i+2}} = 4{,}6692016091...$$

Mandelbrot sets

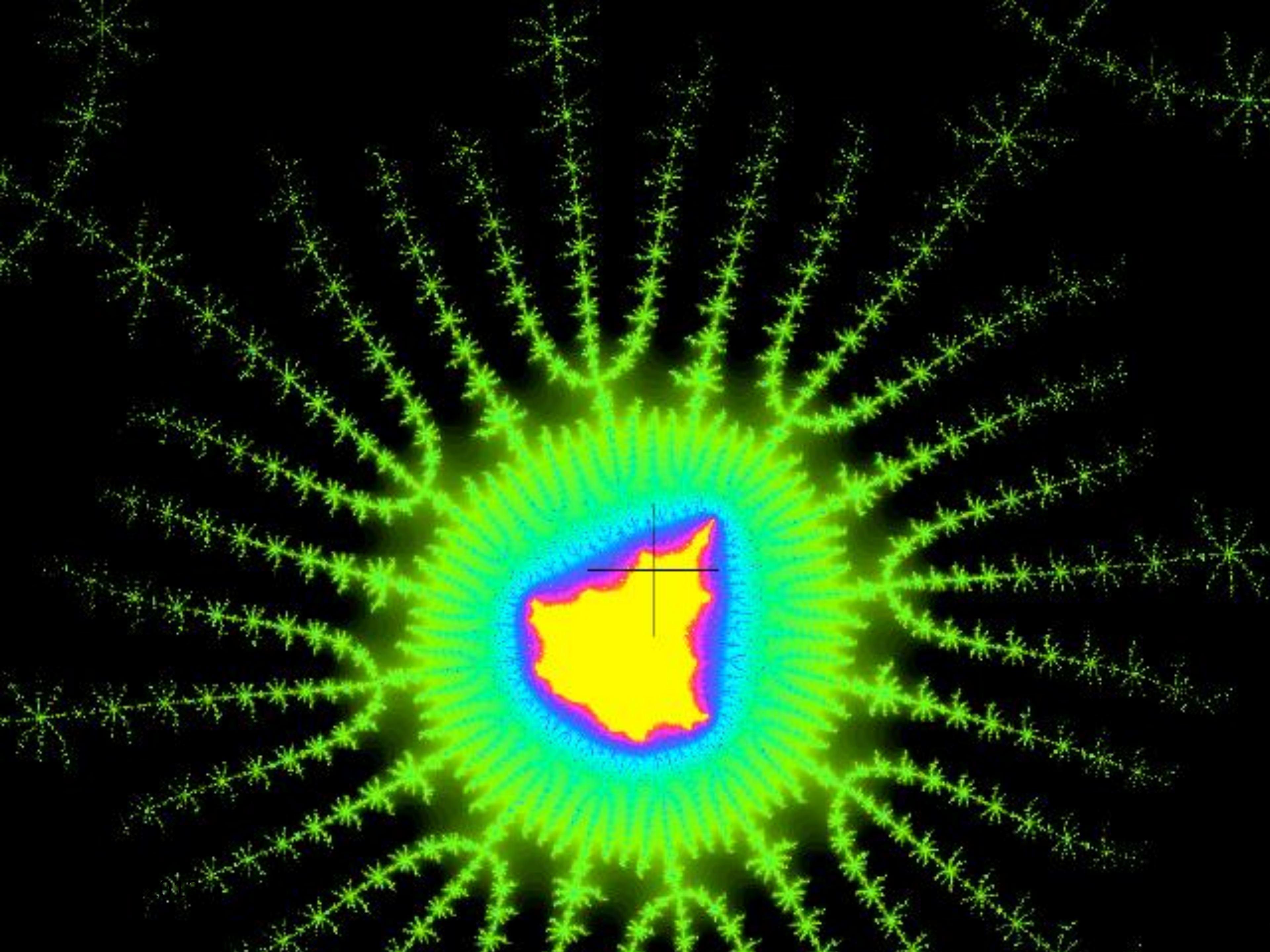
Benoit Mandelbrot (1924 - 2010)

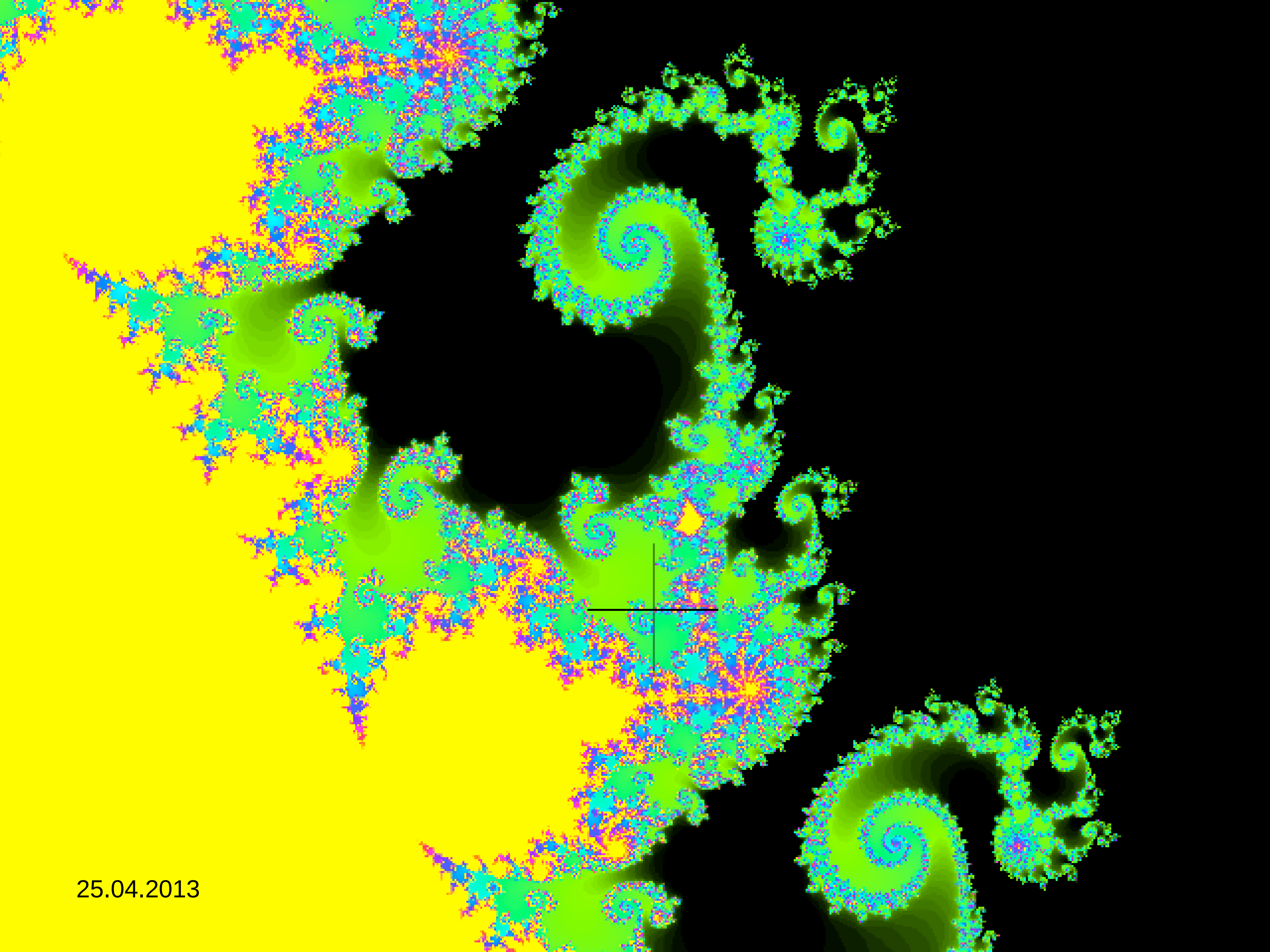
$$z_{n+1} = z_n^2 + c$$

$$c = a + bi$$

$$i = \sqrt{-1}$$

$$z_0 = 0$$





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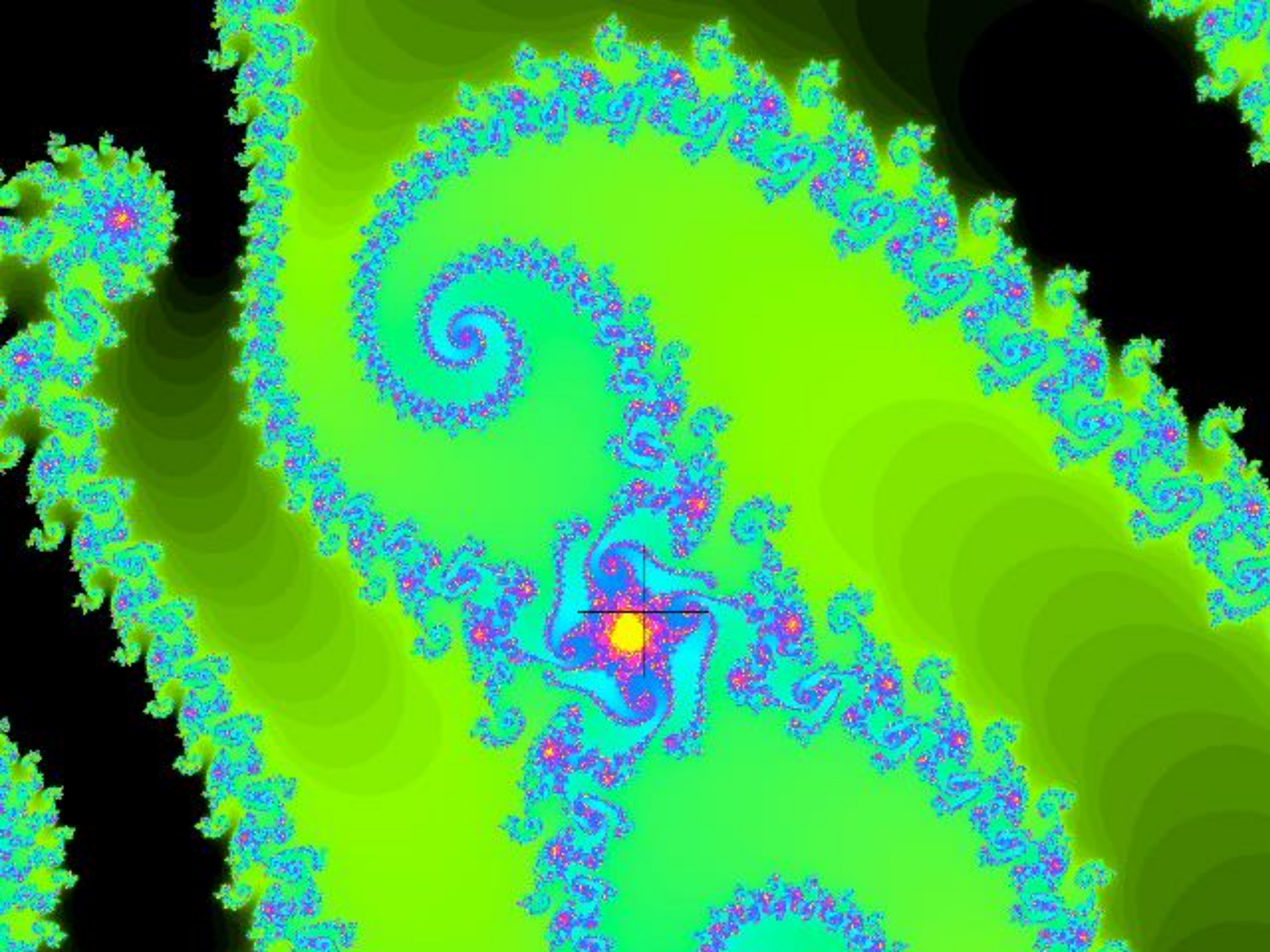

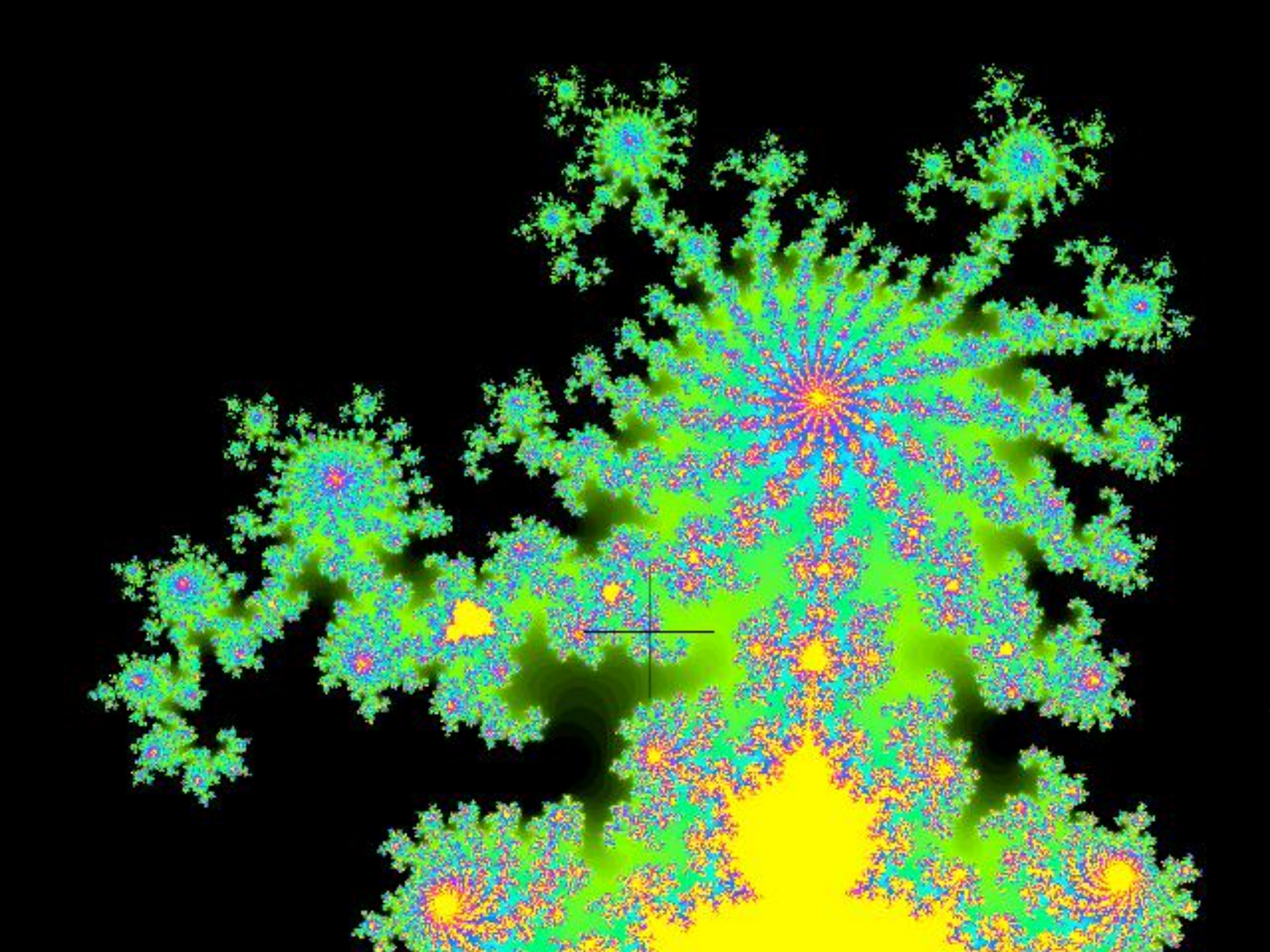

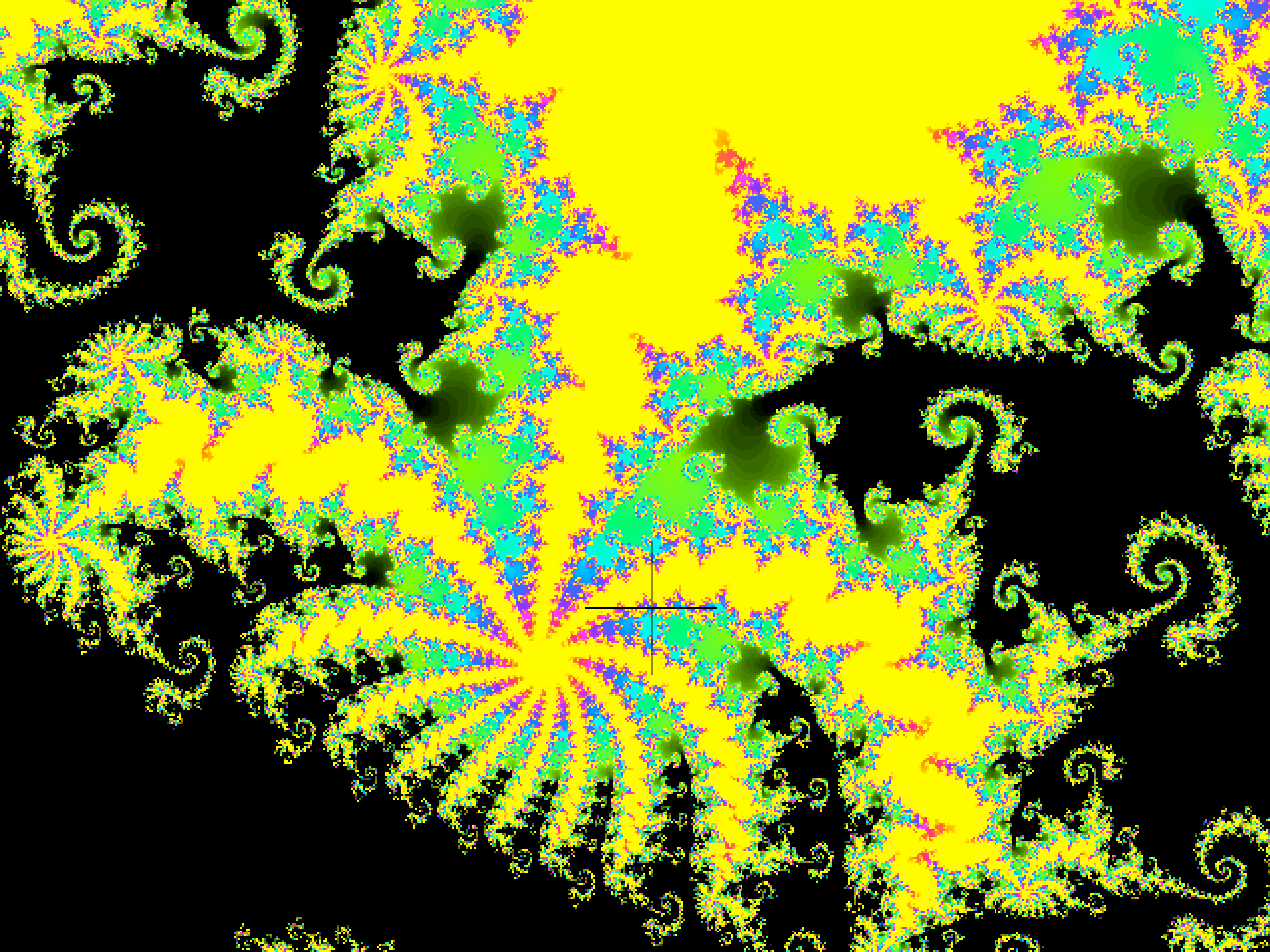

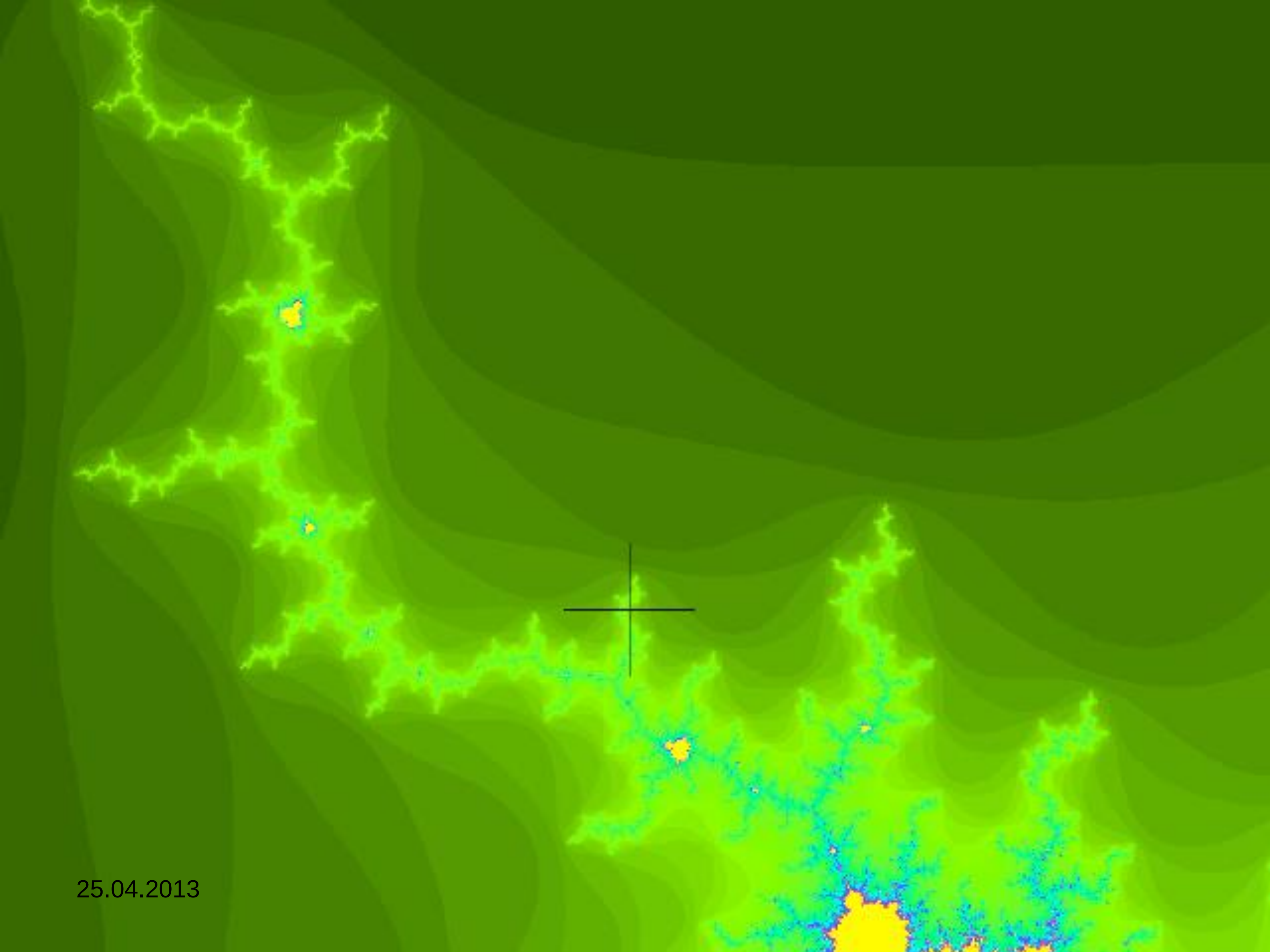

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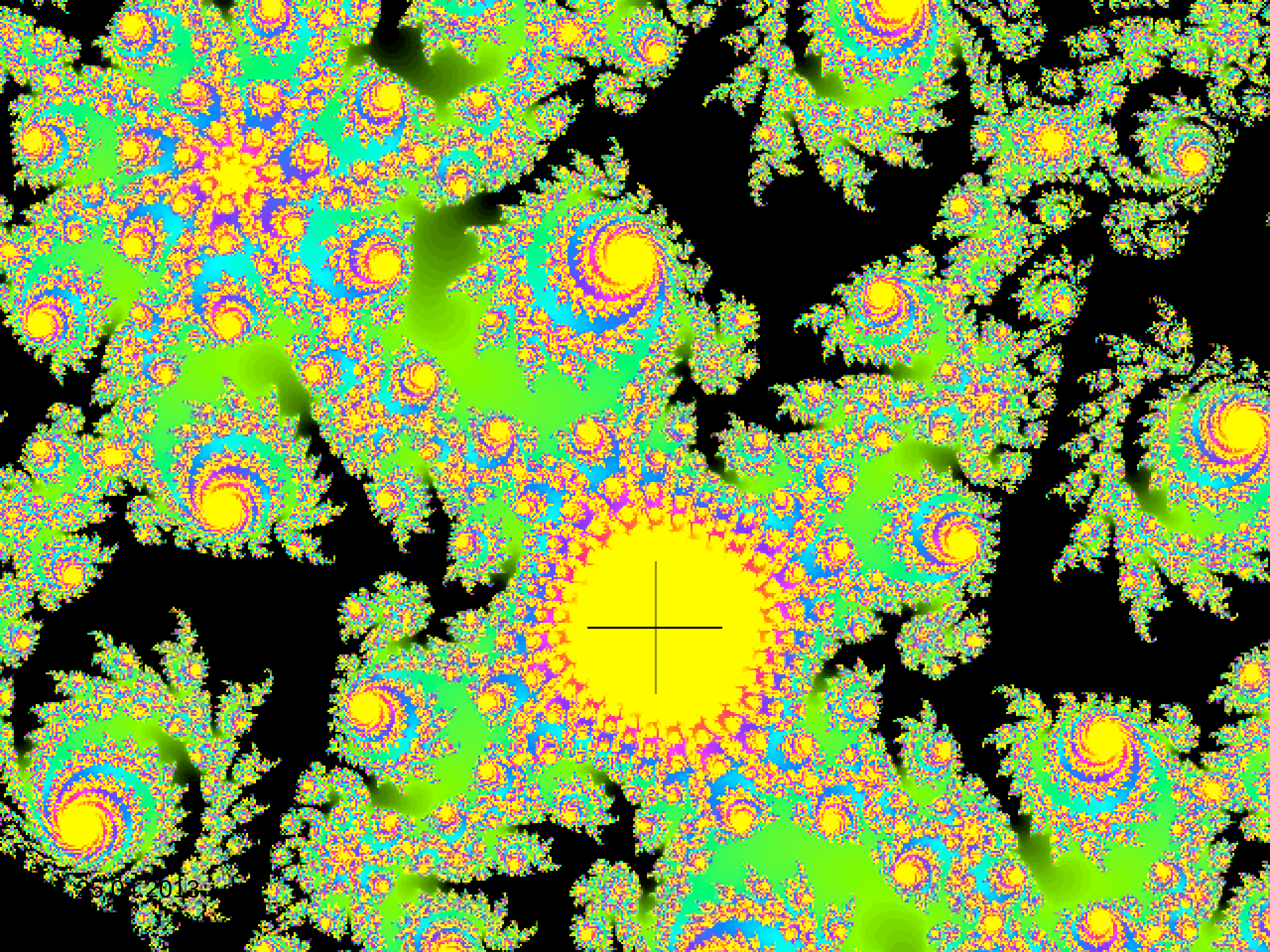
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Conclusion

- Chaos theory has become a branch of mathematics and physics that deals with the behaviour of certain nonlinear dynamical systems
- It has many applications in different fields
- In ScienceDirect – 10 000 papers (from 2009 more than 2500 papers)