See lecture course "Quantum theory of radiation".

Quantum mechanism of the expansion of universe

Abstract

We discuss the hypothesis that quantum processes of the Planck-scale with the Hubble frequency $H \sim 10^{-18}\,\mathrm{sec}^{-1}$ are responsible for the expansion of the universe. A model of these processes is proposed, according to which these processes correspond to zero boson transitions. Using the Lax theory, we show that these transitions lead to fast ($\sim k_B T/\hbar \sim 10^{11}\,\mathrm{sec}^{-1}$) and small ($\sim 10^{-18}$ m) stochastic fluctuations in the volume with a very small ($\sim H$) predominance of processes with increasing volume and with the generation of cosmic background radiation with the correct value of its temperature T.

1 Introduction

According to Hubble's law, our universe expands with a rate per unit length (frequency) $H \approx 2.2 \cdot 10^{-18}\,\mathrm{sec}^{-1}$ (here H^{-1} is the age of the universe). Hubble's expansion is usually explained on the basis of the classical general theory of relativity (see, for example, [1-5]).

However, it is generally accepted that empty space corresponds to the ground (zero-point) state of quantum fields. Thus, the expansion of the universe means an increase in the volume of space occupied by this zero-point field. This provides the basis for the hypothesis that quantum processes should be behind this expansion. In this work, we propose a model of such processes.

In quantum theory, zero-point fluctuations of bosons make a huge contribution to the energy density of the universe $\sim \rho_P = E_P/L_P^3 = 2.9 \times 10^{132}$ eV/m^3, where $E_P = 1.2 \times 10^{28}$ eV and $L_P = 1.6 \times 10^{-35}$ m are Planck energy and length, respectively. This means (provided that the fundamental physical constants do not change with time) that the expansion of the universe causes a huge increase in the zero-point energy of the bosons. This is possible without violation of the energy conservation law, if there is another contribution to the energy of the vacuum, which compensates for this increase. It is usually assumed (see, for example, [6-8]) that the effects of gravity should lead to such compensation: gravitational forces are attractive and they increase with the decreasing distance. This suggests that compensation can occur due to the negative gravitational energy of the vacuum superstructure at the Planck limit [8]. A complete cancellation, possibly with a slight discrepancy, may be the result of zero energy of the initial quantum fluctuation, which, according to [9], spawned the universe.

This article takes the basis of the above assumption that this cancellation is provided by particles with negative gravitational energy of the Planck scale. This point of view is supported by the following consideration of the effect of the gravitational self-action of a wave packet of a particle of small size l, comparable with L_P, on its total energy E [10]. Assuming that gravitational interaction can be considered classically, this energy is determined by the sum of kinetic energy of localization $\sim \hbar c/l$ and the energy $E_G \sim -GE^2/lc^4$ of gravitational self-action, where G is the gravitational constant (consideration is carried out in order of magnitude). This gives the following equation for the total energy $E = \hbar c/l - GE^2/lc^4$ of the wave packet. This equation has two solutions: $E = E_P\left(\pm\sqrt{1+(l/2L_P)^2} - l/2L_P\right)$. The solution with the plus sign corresponds to a wave packet with the positive energy. This energy cannot exceed E_P (a hypothetical particle with $E \approx E_P$ is called a Planck particle, or maxon).

More important in the context is the solution with a minus sign, which, we assume, describes particles with negative energy. For these particles it is energetically beneficial to stay apart: indeed, in volume $l^3 \gg L_P^3$ $\quad N \sim (l/L_P)^3 \gg 1$ non-overlapping wave packets have $\sim (l/L_P)^2 \gg 1$ times larger total negative energy than the single wave packet of the same size. Therefore, it is energetically beneficial to place the aforementioned particles in a space separately from each other. In other words, the considered particles of negative energy are repelled at the Planck limit. The distance between particles L and the size of them L_0 can be determined if to take into account the standard repulsive interaction $\lambda\left(\phi^+\phi\right)^2$ (here $\lambda > 0$, ϕ is the wave function of the quantum field); this interaction works against gravitational collapse of particles, giving $L \approx 2L_P$ and $L_0 \approx L_P$ [10,11].

If such a vacuum structure is true, then the expansion of the universe can occur as quantum processes with the creation of two excitations (particles): one with positive energy close to E_P and another with almost the same but negative energy. The created particles push apart existing particles, which leads to an increase in the size of the universe.

2 The model

To determine the rate of such a process, it is considered here that both, positive and negative energy excitations correspond to bosons with energy $\pm E_0 \cong \pm E_P$ of the zero-point levels. The energy level diagram is shown in Fig. E.1.

The Hamiltonian of the system under consideration has the form

$$\hat{H} = \hat{H}_0(\hat{I} + \hat{\sigma}_z)/2 + \hat{H}_1(\hat{I} - \hat{\sigma}_z)/2 + \hat{H}_{int}, \tag{E.1}$$

where $\hat{H}_0 = H_{0+} + H_{0-}$, $H_{0\pm} = \sum_k \hbar\omega_k\left(\hat{a}^+_{k\pm}\hat{a}_{k\pm} + 1/2\right)$ are the Hamiltonians of bosons with positive and negative zero-point energy, $\hat{a}^+_{k\pm}$ and $\hat{a}_{k\pm}$ are creation and destruction operators of bosons, $\omega_k = ck$ is the frequency of boson with the wave number k,

$$\hat{H}_1 = \hbar\Delta + e^{\nabla}\hat{H}_0 e^{-\nabla} \tag{E.2}$$

is the Hamiltonian of bosons after the transition, $\hbar\Delta = E_1 - E_0$ is the small energy change due to transition between zero-point states (this change is compensated by the energy of the created bosons), $\nabla = \nabla_+ + \nabla_-$ is the shift operator of bosons, $\hat{H}_{int} = V\hat{\sigma}_x$ is the interaction of bosons causing the transitions, $\hat{\sigma}_x$ and $\hat{\sigma}_z$ are the Pauli matrices, $\hat{I}$ is the 2×2 unit matrix.

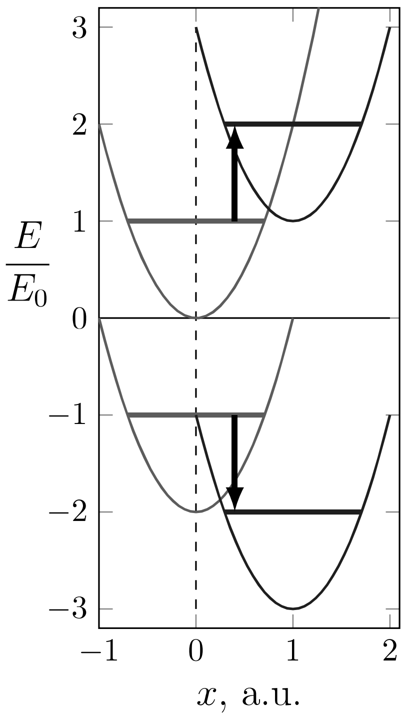

Figure E.1: Energies of the particle (bosons) before and after the transition: the green horizontal lines correspond to the energies of the initial and final states of boson with positive energy, the blue horizontal lines correspond to the initial and final states of boson with negative energy, green and blue parabolas - to the initial and final potential energies of bosons. The thick arrows (up and down) - zero-boson transitions with the energies $\pm E_0 \cong \pm E_P$; x corresponds to generalized coordinate of bosons.

Only spherical modes k are considered. In this case, the shift operator ∇ is the sum $\nabla_\pm = \sum_k \nabla_{k\pm}$ of the shift operators $\nabla_{k\pm} = x_{0k}\partial/\partial x_{k\pm}$ of these modes, where x_{0k} is the shift of the equilibrium position of the mode with the generalized coordinate $x_{k\pm} = \sqrt{\hbar/2\omega_k}(\hat{a}_{k\pm} + \hat{a}^+_{k\pm})$. The density of these modes is $\rho(k) = 3k^2(c/\omega_P)^3$, where $\omega_P = E_p/\hbar$ is the Planck frequency. The spherical displacement $x_{r\pm}$ at the distance r can be represented as the sum of spherical modes x_k as follows:

$$x_{r\pm} = \sqrt{1/4\pi V}\, r^{-1} Im \sum_k k^2 e^{ikr} x_{k\pm}. \quad \text{(E.3)}$$

Here, it is assumed that the quantum transition leads to a finite change in the volume (Δ_V) of the fields of the zero point state. This change can be presented as $\Delta_V = 4\pi R^2 \Delta_R$, where Δ_R is the change of the radius R of the space as the result of the transition. Taking into account Eq. (E.3), one finds $\Delta_R \propto R^{-1} Im \int dk\, k^2 x_{0k} e^{ikR}$. The change of the volume of the zero-point fields Δ_V is finite if $x_{0k} \propto k^{-2}$.

3 The rate of transitions

According to the theory of Lax [12], the rate of a quantum transition with creation of a pair of particles with total energy $E = 0$ can be presented in the form

$$W_+ = |V|^2 \int dt e^{-i\Delta t} F^2(t), \quad \text{(E.4)}$$

where

$$F(t) = \left\langle e^{\nabla_\pm} e^{-\nabla_\pm(t)} \right\rangle \quad \text{(E.5)}$$

is the Fourier transform of the spectrum of bosonic transitions followed by the shift of the equilibrium positions of bosons,

$$\nabla_\pm(t) = \sum_k \sqrt{\omega_k/2\hbar}\, x_{0k} \left(\hat{a}_{k\pm} e^{-i\omega_k t} + \hat{a}^+_{k\pm} e^{i\omega_k t} \right)$$

are the shift operators at time moment t (integrals are taken from $-\infty$ to ∞). Applying Wick's theorem, one obtains [12]

$$F(t) = e^{g(t)},$$

where

$$g(t) = \langle \nabla_\pm (\nabla_\pm - \nabla_\pm(t)) \rangle = \\ = (1/2\hbar) \sum_k \omega_k x_{0k}^2 \left((n_{\omega_k} + 1) \left(e^{i\omega_k t} - 1 \right) + n_{\omega_k} \left(e^{-i\omega_k t} - 1 \right) \right), \tag{E.6}$$

$n_\omega = \left(\exp(\hbar\omega/k_B T) - 1\right)^{-1}$ is the Planck population factor, T is the temperature of cosmic background radiation. Replacing the sum over k by the integral over the wavenumbers according to $\sum_k \to \int dk\, \rho(k)$, and using the relation $n_{-\omega} + 1 = -n_\omega$, we obtain the following simple equation:

$$g(t) = \int d\omega\, \omega^{-1} \left(e^{i\omega t} - 1 \right) (n_\omega + 1), \tag{E.7}$$

where the integral is taken from $-\omega_{\mathrm{P}}$ to ω_{P}. A significant contribution to the integral (E.4) is made by low frequencies $|\omega| \lesssim k_B T/\hbar$, which correspond to zero-boson transitions (ZBT) from a neighbourhood of the initial zero-point states to a neighbourhood of the final zero-point states of bosons. These transitions are analogous to the zero-phonon transitions in optical centers in crystals [13,14]. The contribution of ZBTs is given by large values of $|t| \gg \hbar/k_B T$. To find this contribution, it is necessary to single out in the integral (E.7) the term $k_B T/\hbar\omega^2$ that is divergent for $\omega \to 0$. Then, taking into account the relation $(e^{i\omega t} - 1)/\omega^2 = \pi |t| \left(-\delta(\omega) + i t^{-1} \delta'(\omega) \right)$, $|t| \to \infty$ one gets for large $|t|$

$$g(t) = -\gamma |t| + i\alpha t/|t| - S.$$

Here $\gamma = \pi k_B T$ is the broadening of ZBT, $\alpha = \gamma/2k_B T$ is the asymmetry factor, stemming from the difference of probabilities of Stokes and anti-Stokes transitions [15],

$$S = \int_0^{\omega_P} \frac{d\omega}{\omega} \left(2n_\omega + 1 - \frac{2k_B T}{\hbar\omega} \right) \cong \ln \left(\frac{E_{\mathrm{P}}}{2k_B T} \right) \tag{E.8}$$

is the Huang-Rhys factor determining the probability of ZBT.

Simple calculation gives $W_+ = |V|^2 w$, where

$$w = \left(\frac{2k_B T}{E_P} \right)^2 \frac{\gamma + \alpha\Delta}{\gamma^2 + \Delta^2/4}. \tag{E.9}$$

is the normalized probability of the transition. If $\hbar\Delta \ll k_B T$, then $W_+ = \pi^{-1} |2V/\omega_P|^2 (k_B T + \hbar\Delta/2)$. The factor V, by its physical meaning is the frequency of the attempts to make a transition. Its value is not known. However, one can expect that it has the order ω_P - the only known frequency unit Planck scale. If this is so, then the rates of transitions leading to expansion of universe are given by the equation

$$W_+ \sim k_B T/\hbar + \Delta/2. \tag{E.10}$$

Note that the boson excitation spectra upon transition (given by the Fourier transform of $F(t)$) for small $\omega \lesssim k_B T/\hbar$ are given by a similar to Eq. (E.9) formula

$$I(\omega) = \frac{2k_B T(\gamma + \alpha\omega)}{E_P(\gamma^2 + \omega^2)}.$$

The intensity of these spectra at zero frequency $I(0) \sim \omega_P^{-1}$ is a determining factor in the considered expansion mechanism of the universe. Indeed, one can estimate the probability as $W_+ \sim k_B T \omega_P^2 I(0)^2/\hbar \sim k_B T/\hbar$ in agreement with Eq. (E.10). Here it is taken into account that w is determined by the overlapping integral of $I(\omega)$ and $I(\Delta - \omega)$. For small Δ the overlapping takes place for $|\omega| \lesssim k_B T$.

4 Account for reverse transitions

In addition to the transitions leading to the expansion of the universe it is need to take into account also reverse transitions, which lead to the compression of the universe. The rates of these transitions are given by analogous to Eq. (E.10) formula but with $-\Delta$ instead of Δ. Universe will, then, expand if the rate of the inverse transitions is smaller. This occurs if $\Delta > 0$. In this case the resulting rate of transitions leading to expansion of the universe is given by the equation

$$W = H = W_{+} - W_{-} \sim \Delta. \tag{E.11}$$

Thus, in the proposed theory, the Hubble frequency of expansion of the universe equals to the rate W, and this rate equals to Δ, which determines the gain of the energy of vacuum as a result of the single Planck-scale process causing the expansion of the universe.

5 Discussion

It was shown above that the quantum processes considered here in the universe lead to local volume fluctuations with the frequency $\omega_f \sim k_B T/\hbar \sim 10^{11}$ sec^{-1} and with a small ($\sim H \sim 10^{-18}$ sec^{-1}) predominance of processes with a local increase in volume. The spatial size of fluctuations is $L_f \sim c/\omega_f \sim 1$ cm. Their amplitude A_f can be estimated if we take into account that it consists of $N_f \sim (\omega_f t_P)^{-1} \sim 10^{34}$ individual stochastic processes with an increase and decrease in the volume of the Planck scale. This gives $A_f \sim L_P\sqrt{N_f} \sim 10^{-18}$ m. The small ($\sim H \sim 10^{-18}$) predominance of processes with a local increase of the volume is due to the fact that the energy of the zero-point state of the universe decreases with expansion (in one process this energy decreases by $\hbar H$). This decrease provides kinetic energy for expansion. The total energy is conserved in the processes under consideration.

In the proposed model, the expansion of the universe is accompanied by the birth of bosons, which makes it possible to obtain the following relation for the energy density of the created bosons in the time H^{-1} of the existence of the universe:

$$\rho_b \sim \rho_P \left(k_B T / E_P\right)^4. \tag{E.12}$$

This gives $\rho_b \sim 0.1\rho_c$, where $\rho_c \sim 3H^2/8\pi G \sim 10^9\,\mathrm{eV/m^3}$ is the critical value of the energy density required for the universe to just to halt its expansion. Here it is taken into account that each transition brings an average energy $\sim |\Delta| \sim \hbar H$ of bosons with an average wavelength $L_P\left(E_P/k_B T\right)$, and that the frequency of transitions is $\sim W_{\pm} \sim k_B T/\hbar$. The obtained value of ρ_b is in good agreement with the known energy density of the cosmic background radiation. Thus, the mechanism of quantum expansion of the Universe considered here allows one to explain the temperature of cosmic background radiation $T \sim (0.1\rho_c/\rho_P)^{1/4} T_P$, where $T_P = E_P/k_B$ is the Planck temperature. This result testifies in favour of the validity of the proposed quantum model of the expansion of the universe.

In the model presented here, only massless bosons (photons) were taken into account. The case of bosons with a nonzero rest mass should be considered separately. However, one can assume that for a modern (cold) universe with a temperature of $T = 2.7$ K, one can neglect the processes with the creation and destruction of such bosons.

Only the case of flat space-time was considered here. Nevertheless, our model also allows us to take into account the effects of the general theory of relativity by choosing the correct local time; e.g. the actual space-time geometry can be taken into account by choosing (in appropriate units) $H = 1/2t$ in the case of radiation-dominated expansion or $H = 2/3t$ in the case of matter dominated expansion [1-7].

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