LTAT.03.019 Funktsionaalprogrammeerimine

Tüübituletus

Tüübituletus

- Church'i-stiilis λ-arvutustes sisaldavad λ-termid piisavalt tüübiannotatsioone ning tüübikontroll ja tüübituletus on lihtne.
- Curry-stiilis süsteemides on termid ilma tüübiannotatsioonideta ja tüübid on termi korrektsuse predikaadid.
- Üldjuhul on Curry-stiilis tüübisüsteemides tüübituletus (ja ka tüübikontroll) lahendamatud ülesanded.
  - Teist-järku λ-arvutus.
- Kuid lihtsamate süsteemide jaoks on tüübituletus võimalik.
  - Lihtsalt tüübitud λ-arvutus.
  - Hindley-Milner'i polümorfism.

Curry-stiilis lihtsalt tüübitud λ-arvutus

- Tüübid (sama kui $\lambda\!\rightarrow$ a'la Church):

$$\begin{array}{lcll} \tau & ::= & \alpha & \text{tüübimuutuja} \\ & | & \tau_1 \rightarrow \tau_2 & \text{funktsioonitüüp} \end{array}$$

- Termid (sama kui puhtas λ-arvutuses):

$$\begin{array}{lcll} e & ::= & x & \text{muutuja} \\ & | & e_1\ e_2 & \text{aplikatsioon} \\ & | & \lambda x.\, e & \text{abstraktsioon} \end{array}$$

- Tüüpimisreeglid:

$$\frac{}{\Gamma, x : \tau \vdash x : \tau} \qquad \frac{\Gamma, x : \sigma \vdash e : \tau}{\Gamma \vdash \lambda x.\, e : \sigma \rightarrow \tau}$$

$$\frac{\Gamma \vdash e_1 : \sigma \rightarrow \tau \quad \Gamma \vdash e_2 : \sigma}{\Gamma \vdash e_1\ e_2 : \tau}$$

Curry-stiilis lihtsalt tüübitud λ-arvutus

- Tähistused:
  - $S, S', \ldots$ tüübisubstitutsioonid
  - $\tau \succ \tau' \iff \exists S\,[\tau' = S(\tau)]$;
  - $\Gamma \succ \Gamma' \iff \exists S\,[\Gamma' \supseteq S(\Gamma)]$.
- Definitsioon: (Γ, τ) on termi e printsipiaalne paar parajasti siis, kui
  (i) $\Gamma \vdash e : \tau$;
  (ii) $\Gamma' \vdash e : \tau' \iff \Gamma \succ \Gamma' \ \wedge\ \tau \succ \tau'$.
- Kinnise termi e printsipiaalses paaris $(\emptyset, \tau)$ olevat tüüpi τ nimetatakse printsipiaalseks tüübiks.
- Teoreem: Iga tüübitava termi e jaoks leidub printsipiaalne paar (Γ, τ). See paar on unikaalne tüübimuutujate ümbernimetamise täpsuseni.

Tüübituletus $\lambda\!\to$ a'la Curry jaoks

Tüübituletuse algoritm:

- Annoteerida iga alamavaldis ja muutuja siduvesinemine unikaalse tüübimuutujaga.
- Genereeri kitsenduste süsteem kasutades järgnevaid reegleid:

$$\frac{x^\alpha \in \Gamma}{\Gamma \vdash x^\beta \Rightarrow \{\alpha = \beta\}} \qquad \frac{\Gamma, x^\alpha \vdash e^\beta \Rightarrow E}{\Gamma \vdash (\lambda x^\alpha .\, e^\beta)^\gamma \Rightarrow \{\gamma = \alpha \to \beta\} \cup E}$$

$$\frac{\Gamma \vdash e_1^\alpha \Rightarrow E_1 \qquad \Gamma \vdash e_2^\beta \Rightarrow E_2}{\Gamma \vdash (e_1^\alpha\, e_2^\beta)^\gamma \Rightarrow \{\alpha = \beta \to \gamma\} \cup E_1 \cup E_2}$$

- Lahenda kitsenduste süsteem leides selle kõige üldisema unifitseerija.
  - Kui seda ei leidu $\Longrightarrow$ term ei ole tüübitav.

Tüübituletus $\lambda{\to}$ a'la Curry jaoks

$$\dfrac{}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5}z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}}$$

Tüübituletus $\lambda\!\to$ a'la Curry jaoks

$$\dfrac{\dfrac{}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2}}}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}}$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1}}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2}}}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}}$$

Tüübituletus $\lambda\!\to$ a'la Curry jaoks

$$
\cfrac{\cfrac{\cfrac{\cfrac{}{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2}}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1}}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2}}}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}}
$$

Tüübituletus $\lambda\!\to$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash x^{\alpha_4}} \qquad \dfrac{}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1}}}{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2}}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1}}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2}}}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}}$$

Tüübituletus $\lambda\!\to$ a'la Curry jaoks

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\overline{\Gamma \vdash x^{\alpha_4}} \qquad \dfrac{\overline{\Gamma \vdash y^{\alpha_5}} \qquad \overline{\Gamma \vdash z^{\alpha_6}}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1}}}{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2}}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1}}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2}}}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}}}{}
$$

Tüübituletus $\lambda\!\to$ a'la Curry jaoks

$$
\dfrac{\dfrac{\Gamma \vdash x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \quad \dfrac{\overline{\Gamma \vdash y^{\alpha_5}} \quad \overline{\Gamma \vdash z^{\alpha_6}}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1}}}{\dfrac{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2}}{\dfrac{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1}}{\dfrac{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2}}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}}}}}
$$

Kitsendused:

$$E_1 = \{\alpha_1 = \alpha_4\}$$

Tüübituletus $\lambda\!\rightarrow$ a'la Curry jaoks

$$
\dfrac{
\dfrac{
\dfrac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1}
\qquad
\dfrac{
\dfrac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2}
\qquad
\dfrac{}{\Gamma \vdash z^{\alpha_6}}
}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1}}
}{
\dfrac{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2}}{
\dfrac{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1}}{
x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2}
}}}
}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}}
$$

Kitsendused:

$$
\begin{aligned}
E_1 &= \{\alpha_1 = \alpha_4\} \\
E_2 &= \{\alpha_2 = \alpha_5\}
\end{aligned}
$$

Tüübituletus $\lambda\!\rightarrow$ a'la Curry jaoks

$$
\dfrac{
\dfrac{\dfrac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1}
\quad
\dfrac{\dfrac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \dfrac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1}}}
{\dfrac{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2}}
{\dfrac{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1}}
{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2}}}}
}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}}
$$

Kitsendused:

$$
\begin{aligned}
E_1 &= \{\alpha_1 = \alpha_4\} \\
E_2 &= \{\alpha_2 = \alpha_5\} \\
E_3 &= \{\alpha_3 = \alpha_6\}
\end{aligned}
$$

Tüübituletus $\lambda\!\rightarrow$ a'la Curry jaoks

$$
\dfrac{
  \dfrac{
    \dfrac{
      \dfrac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \;\Rightarrow\; E_1}
      \qquad
      \dfrac{
        \dfrac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \;\Rightarrow\; E_2}
        \quad
        \dfrac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \;\Rightarrow\; E_3}
      }{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \;\Rightarrow\; E_4}
    }{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2}}
  }{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1}}
}{
  \dfrac{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2}}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}}
}
$$

Kitsendused:

$$
\begin{aligned}
E_1 &= \{\alpha_1 = \alpha_4\} \\
E_2 &= \{\alpha_2 = \alpha_5\} \\
E_3 &= \{\alpha_3 = \alpha_6\} \\
E_4 &= \{\alpha_5 = \alpha_6 \rightarrow \beta_1\} \\
    &\quad\; \cup E_2 \cup E_3
\end{aligned}
$$

Tüübituletus $\lambda\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \quad \dfrac{\dfrac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \dfrac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4}}{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \Rightarrow E_5}}{\dfrac{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1}}{\dfrac{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2}}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}}}}$$

Kitsendused:

$$\begin{aligned}
E_1 &= \{\alpha_1 = \alpha_4\} \\
E_2 &= \{\alpha_2 = \alpha_5\} \\
E_3 &= \{\alpha_3 = \alpha_6\} \\
E_4 &= \{\alpha_5 = \alpha_6 \to \beta_1\} \\
&\quad \cup E_2 \cup E_3
\end{aligned}$$

$$E_5 = \{\alpha_4 = \beta_1 \to \beta_2\} \cup E_1 \cup E_4$$

Tüübituletus $\lambda\!\rightarrow$ a'la Curry jaoks

$$
\dfrac{
  \dfrac{
    \dfrac{
      \dfrac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1}
      \quad
      \dfrac{
        \dfrac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2}
        \quad
        \dfrac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}
      }{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4}
    }{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \Rightarrow E_5}
  }{
    \dfrac{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1} \Rightarrow E_6}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2}}
  }
}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}}
$$

Kitsendused:

$$
\begin{aligned}
E_1 &= \{\alpha_1 = \alpha_4\} & E_5 &= \{\alpha_4 = \beta_1 \rightarrow \beta_2\} \cup E_1 \cup E_4 \\
E_2 &= \{\alpha_2 = \alpha_5\} & E_6 &= \{\gamma_1 = \alpha_3 \rightarrow \beta_2\} \cup E_5 \\
E_3 &= \{\alpha_3 = \alpha_6\} & & \\
E_4 &= \{\alpha_5 = \alpha_6 \rightarrow \beta_1\} & & \\
&\quad \cup E_2 \cup E_3 & &
\end{aligned}
$$

Tüübituletus $\lambda\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{x^{\alpha_1}\in\Gamma}{\Gamma\vdash x^{\alpha_4}\Rightarrow E_1}\qquad \dfrac{\dfrac{y^{\alpha_2}\in\Gamma}{\Gamma\vdash y^{\alpha_5}\Rightarrow E_2}\qquad \dfrac{z^{\alpha_3}\in\Gamma}{\Gamma\vdash z^{\alpha_6}\Rightarrow E_3}}{\Gamma\vdash (y^{\alpha_5}z^{\alpha_6})^{\beta_1}\Rightarrow E_4}}{\dfrac{x^{\alpha_1},y^{\alpha_2},z^{\alpha_3}\vdash (x^{\alpha_4}(y^{\alpha_5}z^{\alpha_6})^{\beta_1})^{\beta_2}\Rightarrow E_5}{\dfrac{x^{\alpha_1},y^{\alpha_2}\vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5}z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1}\Rightarrow E_6}{\dfrac{x^{\alpha_1}\vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5}z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2}\Rightarrow E_7}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5}z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}}}}}$$

Kitsendused:

$$\begin{aligned}
E_1 &= \{\alpha_1=\alpha_4\}\\
E_2 &= \{\alpha_2=\alpha_5\}\\
E_3 &= \{\alpha_3=\alpha_6\}\\
E_4 &= \{\alpha_5=\alpha_6\to\beta_1\}\\
&\quad \cup\, E_2\cup E_3
\end{aligned}$$

$$\begin{aligned}
E_5 &= \{\alpha_4=\beta_1\to\beta_2\}\cup E_1\cup E_4\\
E_6 &= \{\gamma_1=\alpha_3\to\beta_2\}\cup E_5\\
E_7 &= \{\gamma_2=\alpha_2\to\gamma_1\}\cup E_6
\end{aligned}$$

Tüübituletus $\lambda\to$ a'la Curry jaoks

$$
\dfrac{\dfrac{x^{\alpha_1}\in\Gamma}{\Gamma\vdash x^{\alpha_4}\Rightarrow E_1} \quad \dfrac{\dfrac{y^{\alpha_2}\in\Gamma}{\Gamma\vdash y^{\alpha_5}\Rightarrow E_2} \quad \dfrac{z^{\alpha_3}\in\Gamma}{\Gamma\vdash z^{\alpha_6}\Rightarrow E_3}}{\Gamma\vdash (y^{\alpha_5}z^{\alpha_6})^{\beta_1}\Rightarrow E_4}}{\dfrac{x^{\alpha_1},y^{\alpha_2},z^{\alpha_3}\vdash (x^{\alpha_4}(y^{\alpha_5}z^{\alpha_6})^{\beta_1})^{\beta_2}\Rightarrow E_5}{\dfrac{x^{\alpha_1},y^{\alpha_2}\vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5}z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1}\Rightarrow E_6}{\dfrac{x^{\alpha_1}\vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5}z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2}\Rightarrow E_7}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5}z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}\Rightarrow E_8}}}}
$$

Kitsendused:

$$
\begin{aligned}
E_1 &= \{\alpha_1=\alpha_4\} & E_5 &= \{\alpha_4=\beta_1\to\beta_2\}\cup E_1\cup E_4\\
E_2 &= \{\alpha_2=\alpha_5\} & E_6 &= \{\gamma_1=\alpha_3\to\beta_2\}\cup E_5\\
E_3 &= \{\alpha_3=\alpha_6\} & E_7 &= \{\gamma_2=\alpha_2\to\gamma_1\}\cup E_6\\
E_4 &= \{\alpha_5=\alpha_6\to\beta_1\} & E_8 &= \{\gamma_3=\alpha_1\to\gamma_2\}\cup E_7\\
&\quad \cup E_2\cup E_3 & &
\end{aligned}
$$

Tüübituletus $\lambda\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \quad \dfrac{\dfrac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \dfrac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4}}{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \Rightarrow E_5}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1} \Rightarrow E_6}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2} \Rightarrow E_7}}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3} \Rightarrow E_8}$$

Kitsendused:

$$\begin{aligned} E_8 \quad = \quad & \{\alpha_1 = \alpha_4,\ \alpha_2 = \alpha_5,\ \alpha_3 = \alpha_6, \\ & \ \alpha_5 = \alpha_6 \rightarrow \beta_1,\ \alpha_4 = \beta_1 \rightarrow \beta_2, \\ & \ \gamma_1 = \alpha_3 \rightarrow \beta_2, \\ & \ \gamma_2 = \alpha_2 \rightarrow \gamma_1, \\ & \ \gamma_3 = \alpha_1 \rightarrow \gamma_2\} \end{aligned}$$

Tüübituletus $\lambda\!\rightarrow$ a'la Curry jaoks

$$
\dfrac{\dfrac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \quad \dfrac{\dfrac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \dfrac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4}}{\dfrac{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \Rightarrow E_5}{\dfrac{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1} \Rightarrow E_6}{\dfrac{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2} \Rightarrow E_7}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3} \Rightarrow E_8}}}}
$$

Kitsendused:

$$
\begin{aligned}
E_8 \quad = \quad \{ & \alpha_1 = \alpha_4,\ \alpha_2 = \alpha_5,\ \alpha_3 = \alpha_6, \\
& \alpha_5 = \alpha_6 \rightarrow \beta_1,\ \alpha_4 = \beta_1 \rightarrow \beta_2, \\
& \gamma_1 = \alpha_3 \rightarrow \beta_2, \\
& \gamma_2 = \alpha_2 \rightarrow \gamma_1, \\
& \gamma_3 = \alpha_4 \rightarrow \gamma_2 \}
\end{aligned}
$$

Tüübituletus $\lambda\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \qquad \dfrac{\dfrac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \dfrac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4}}{\dfrac{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \Rightarrow E_5}{\dfrac{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1} \Rightarrow E_6}{\dfrac{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2} \Rightarrow E_7}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3} \Rightarrow E_8}}}}$$

Kitsendused:

$$\begin{aligned} E_8 \quad = \quad \{ & \alpha_1 = \alpha_4,\ \alpha_2 = \alpha_5,\ \alpha_3 = \alpha_6, \\ & \alpha_5 = \alpha_6 \rightarrow \beta_1,\ \alpha_4 = \beta_1 \rightarrow \beta_2, \\ & \gamma_1 = \alpha_3 \rightarrow \beta_2, \\ & \gamma_2 = \alpha_5 \rightarrow \gamma_1, \\ & \gamma_3 = \alpha_4 \rightarrow \gamma_2 \} \end{aligned}$$

Tüübituletus $\lambda\!\to$ a'la Curry jaoks

$$
\dfrac{
  \dfrac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1}
  \qquad
  \dfrac{
    \dfrac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2}
    \quad
    \dfrac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}
  }{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4}
}{
  \dfrac{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \Rightarrow E_5}{
    \dfrac{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1} \Rightarrow E_6}{
      \dfrac{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2} \Rightarrow E_7}{
        \vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3} \Rightarrow E_8
      }
    }
  }
}
$$

Kitsendused:

$$
\begin{aligned}
E_8 \quad = \quad & \{\alpha_1 = \alpha_4,\ \alpha_2 = \alpha_5,\ \alpha_3 = \alpha_6, \\
& \alpha_5 = \alpha_6 \to \beta_1,\ \alpha_4 = \beta_1 \to \beta_2, \\
& \gamma_1 = \alpha_6 \to \beta_2, \\
& \gamma_2 = \alpha_5 \to \gamma_1, \\
& \gamma_3 = \alpha_4 \to \gamma_2\}
\end{aligned}
$$

Tüübituletus $\lambda\!\to$ a'la Curry jaoks

$$
\dfrac{\dfrac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \quad \dfrac{\dfrac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \dfrac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4}}{\dfrac{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \Rightarrow E_5}{\dfrac{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1} \Rightarrow E_6}{\dfrac{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2} \Rightarrow E_7}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3} \Rightarrow E_8}}}}
$$

Kitsendused:

$$
\begin{aligned}
E_8 \quad = \quad \{ & \alpha_1 = \alpha_4,\ \alpha_2 = \alpha_5,\ \alpha_3 = \alpha_6, \\
& \alpha_5 = \alpha_6 \to \beta_1,\ \alpha_4 = \beta_1 \to \beta_2, \\
& \gamma_1 = \alpha_6 \to \beta_2, \\
& \gamma_2 = (\alpha_6 \to \beta_1) \to \gamma_1, \\
& \gamma_3 = \alpha_4 \to \gamma_2 \}
\end{aligned}
$$

Tüübituletus $\lambda\!\rightarrow$ a'la Curry jaoks

$$
\dfrac{
\dfrac{
\dfrac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1}
\qquad
\dfrac{
\dfrac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2}
\quad
\dfrac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}
}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4}
}{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \Rightarrow E_5}
}{
\dfrac{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1} \Rightarrow E_6}{
\dfrac{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2} \Rightarrow E_7}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3} \Rightarrow E_8}}
}
$$

Kitsendused:

$$
\begin{aligned}
E_8 \quad = \quad & \{\alpha_1 = \alpha_4,\ \alpha_2 = \alpha_5,\ \alpha_3 = \alpha_6, \\
& \alpha_5 = \alpha_6 \rightarrow \beta_1,\ \alpha_4 = \beta_1 \rightarrow \beta_2, \\
& \gamma_1 = \alpha_6 \rightarrow \beta_2, \\
& \gamma_2 = (\alpha_6 \rightarrow \beta_1) \rightarrow \gamma_1, \\
& \gamma_3 = (\beta_1 \rightarrow \beta_2) \rightarrow \gamma_2\}
\end{aligned}
$$

Tüübituletus $\lambda\!\to$ a'la Curry jaoks

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \quad \dfrac{\dfrac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \dfrac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4}}{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \Rightarrow E_5}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1} \Rightarrow E_6}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2} \Rightarrow E_7}}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3} \Rightarrow E_8}
$$

Kitsendused:

$$
\begin{aligned}
E_8 \;=\; \{ & \alpha_1 = \alpha_4,\ \alpha_2 = \alpha_5,\ \alpha_3 = \alpha_6, \\
& \alpha_5 = \alpha_6 \to \beta_1,\ \alpha_4 = \beta_1 \to \beta_2, \\
& \gamma_1 = \alpha_6 \to \beta_2, \\
& \gamma_2 = (\alpha_6 \to \beta_1) \to \alpha_6 \to \beta_2, \\
& \gamma_3 = (\beta_1 \to \beta_2) \to \gamma_2 \}
\end{aligned}
$$

Tüübituletus $\lambda\!\rightarrow$ a'la Curry jaoks

$$
\dfrac{
  \dfrac{
    \dfrac{
      \dfrac{
        \dfrac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1}
        \qquad
        \dfrac{
          \dfrac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2}
          \quad
          \dfrac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}
        }{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4}
      }{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \Rightarrow E_5}
    }{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1} \Rightarrow E_6}
  }{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2} \Rightarrow E_7}
}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3} \Rightarrow E_8}
$$

Kitsendused:

$$
\begin{aligned}
E_8 \quad = \quad \{ & \alpha_1 = \alpha_4,\ \alpha_2 = \alpha_5,\ \alpha_3 = \alpha_6, \\
& \alpha_5 = \alpha_6 \to \beta_1,\ \alpha_4 = \beta_1 \to \beta_2, \\
& \gamma_1 = \alpha_6 \to \beta_2, \\
& \gamma_2 = (\alpha_6 \to \beta_1) \to \alpha_6 \to \beta_2, \\
& \gamma_3 = (\beta_1 \to \beta_2) \to (\alpha_6 \to \beta_1) \to \alpha_6 \to \beta_2 \}
\end{aligned}
$$

Tüübituletus $\lambda{\rightarrow}$ a'la Curry jaoks

$$
\dfrac{
  \dfrac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1}
  \qquad
  \dfrac{
    \dfrac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2}
    \quad
    \dfrac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}
  }{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4}
}{
  \dfrac{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \Rightarrow E_5}{
    \dfrac{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1} \Rightarrow E_6}{
      \dfrac{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2} \Rightarrow E_7}{
        \vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3} \Rightarrow E_8
      }
    }
  }
}
$$

Tuletatud tüüp:

$$\gamma_3 \quad = \quad (\beta \to \gamma) \to (\alpha \to \beta) \to \alpha \to \gamma$$

Võrrandite lahendamine unifitseerimisega

- Võrrandeid saab lahendada lihtsustusreeglite korduva rakendamise kaudu.
- Kaks peamist reeglit:
  - asenda võrdus kujul $\tau_1 \to \tau_2 = \tau_3 \to \tau_4$ kahe võrdusega $\tau_1 = \tau_3$ ja $\tau_2 = \tau_4$;
  - olgu võrdus kujul $\alpha = \tau$. Kui $\alpha \in \mathrm{FV}(\tau)$ siis raporteeri viga, vastasel korral asenda kõigis võrdustes α asemel τ.
- Abireeglid:
  - eemalda reeglid kujul $\alpha = \alpha$, $\mathrm{Bool} = \mathrm{Bool}$, jne.;
  - asenda $\tau = \alpha$ võrdusega $\alpha = \tau$;
  - kui leidub võrdus $\tau_1 = \tau_2$, kus peatüübikonstruktorid on erinevad (näit. $\mathrm{Bool} = \alpha_1 \to \alpha_2$), siis raprorteeri viga.

Type inference for $\lambda\!\rightarrow$ a'la Curry

$$\frac{}{\vdash ((\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}(\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6})^{\alpha_7}}$$

Type inference for $\lambda\!\rightarrow$ a'la Curry

$$\dfrac{\dfrac{}{\vdash (\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}} \qquad \dfrac{}{\vdash (\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6}}}{\vdash ((\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}(\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6})^{\alpha_7}}$$

Type inference for $\lambda\!\!\rightarrow$ a'la Curry

$$\dfrac{\dfrac{\dfrac{}{x^{\alpha_1} \vdash x^{\alpha_2}}}{\vdash (\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}} \qquad \dfrac{\dfrac{}{x^{\alpha_4} \vdash x^{\alpha_5}}}{\vdash (\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6}}}{\vdash ((\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}(\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6})^{\alpha_7}}$$

Type inference for $\lambda\!\rightarrow$ a'la Curry

$$\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1}}{\vdash (\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}} \qquad \dfrac{\dfrac{x^{\alpha_4} \in \{x^{\alpha_4}\}}{x^{\alpha_4} \vdash x^{\alpha_5} \Rightarrow E_2}}{\vdash (\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6}}}{\vdash ((\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}(\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6})^{\alpha_7}}$$

Kitsendused:

$$\begin{aligned} E_1 &= \{\alpha_1 = \alpha_2\} \\ E_2 &= \{\alpha_4 = \alpha_5\} \end{aligned}$$

Type inference for $\lambda\!\rightarrow$ a'la Curry

$$\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1}}{\vdash (\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3} \Rightarrow E_3} \qquad \dfrac{\dfrac{x^{\alpha_4} \in \{x^{\alpha_4}\}}{x^{\alpha_4} \vdash x^{\alpha_5} \Rightarrow E_2}}{\vdash (\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6} \Rightarrow E_4}}{\vdash ((\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}(\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6})^{\alpha_7}}$$

Kitsendused:

$$\begin{array}{lcl} E_1 & = & \{\alpha_1 = \alpha_2\} \\ E_2 & = & \{\alpha_4 = \alpha_5\} \\ E_3 & = & \{\alpha_3 = \alpha_1 \to \alpha_2\} \cup E_1 \\ E_4 & = & \{\alpha_6 = \alpha_4 \to \alpha_5\} \cup E_2 \end{array}$$

Type inference for $\lambda\!\rightarrow$ a'la Curry

$$
\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1}}{\vdash (\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3} \Rightarrow E_3} \qquad \dfrac{\dfrac{x^{\alpha_4} \in \{x^{\alpha_4}\}}{x^{\alpha_4} \vdash x^{\alpha_5} \Rightarrow E_2}}{\vdash (\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6} \Rightarrow E_4}}{\vdash ((\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}(\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6})^{\alpha_7} \Rightarrow E_5}
$$

Kitsendused:

$$
\begin{array}{lcl}
E_1 & = & \{\alpha_1 = \alpha_2\} \\
E_2 & = & \{\alpha_4 = \alpha_5\} \\
E_3 & = & \{\alpha_3 = \alpha_1 \rightarrow \alpha_2\} \cup E_1 \\
E_4 & = & \{\alpha_6 = \alpha_4 \rightarrow \alpha_5\} \cup E_2 \\
E_5 & = & \{\alpha_3 = \alpha_6 \rightarrow \alpha_7\} \cup E_3 \cup E_4
\end{array}
$$

Type inference for $\lambda\!\!\rightarrow$ a'la Curry

$$\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1}}{\vdash (\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3} \Rightarrow E_3} \qquad \dfrac{\dfrac{x^{\alpha_4} \in \{x^{\alpha_4}\}}{x^{\alpha_4} \vdash x^{\alpha_5} \Rightarrow E_2}}{\vdash (\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6} \Rightarrow E_4}}{\vdash ((\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}(\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6})^{\alpha_7} \Rightarrow E_5}$$

Kitsendused:

$$\begin{aligned} E_5 \quad = \quad \{ & \alpha_1 = \alpha_2,\ \alpha_4 = \alpha_5, \\ & \alpha_3 = \alpha_1 \rightarrow \alpha_2, \\ & \alpha_6 = \alpha_4 \rightarrow \alpha_5, \\ & \alpha_3 = \alpha_6 \rightarrow \alpha_7 \} \end{aligned}$$

Type inference for $\lambda\!\rightarrow$ a'la Curry

$$
\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1}}{\vdash (\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3} \Rightarrow E_3} \qquad \dfrac{\dfrac{x^{\alpha_4} \in \{x^{\alpha_4}\}}{x^{\alpha_4} \vdash x^{\alpha_5} \Rightarrow E_2}}{\vdash (\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6} \Rightarrow E_4}}{\vdash ((\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}(\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6})^{\alpha_7} \Rightarrow E_5}
$$

Kitsendused:

$$
\begin{aligned}
E_5 \quad = \quad \{ \qquad\qquad & \alpha_4 = \alpha_5, \\
\alpha_3 &= \alpha_2 \to \alpha_2, \\
\alpha_6 &= \alpha_4 \to \alpha_5, \\
\alpha_3 &= \alpha_6 \to \alpha_7\}
\end{aligned}
$$

Type inference for $\lambda\!\rightarrow$ a'la Curry

$$\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1}}{\vdash (\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3} \Rightarrow E_3} \qquad \dfrac{\dfrac{x^{\alpha_4} \in \{x^{\alpha_4}\}}{x^{\alpha_4} \vdash x^{\alpha_5} \Rightarrow E_2}}{\vdash (\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6} \Rightarrow E_4}}{\vdash ((\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}(\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6})^{\alpha_7} \Rightarrow E_5}$$

Kitsendused:

$$\begin{aligned} E_5 \quad = \quad \{ & \\ & \alpha_3 = \alpha_2 \rightarrow \alpha_2, \\ & \alpha_6 = \alpha_5 \rightarrow \alpha_5, \\ & \alpha_3 = \alpha_6 \rightarrow \alpha_7 \} \end{aligned}$$

Type inference for $\lambda\!\to$ a'la Curry

$$\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1}}{\vdash (\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3} \Rightarrow E_3} \qquad \dfrac{\dfrac{x^{\alpha_4} \in \{x^{\alpha_4}\}}{x^{\alpha_4} \vdash x^{\alpha_5} \Rightarrow E_2}}{\vdash (\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6} \Rightarrow E_4}}{\vdash ((\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}(\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6})^{\alpha_7} \Rightarrow E_5}$$

Kitsendused:

$$\begin{aligned} E_5 \;=\; \{ & \\ & \alpha_6 = \alpha_5 \to \alpha_5, \\ & \alpha_2 \to \alpha_2 = \alpha_6 \to \alpha_7\} \end{aligned}$$

Type inference for $\lambda\!\rightarrow$ a'la Curry

$$
\dfrac{
\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1}}{\vdash (\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3} \Rightarrow E_3}
\qquad
\dfrac{\dfrac{x^{\alpha_4} \in \{x^{\alpha_4}\}}{x^{\alpha_4} \vdash x^{\alpha_5} \Rightarrow E_2}}{\vdash (\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6} \Rightarrow E_4}
}{
\vdash ((\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}(\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6})^{\alpha_7} \Rightarrow E_5
}
$$

Kitsendused:

$$
\begin{aligned}
E_5 \quad = \quad \{ & \\
& \alpha_2 = \alpha_6, \\
& \alpha_6 = \alpha_5 \rightarrow \alpha_5, \\
& \alpha_2 = \alpha_7 \}
\end{aligned}
$$

Type inference for $\lambda\!\rightarrow$ a'la Curry

$$\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1}}{\vdash (\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3} \Rightarrow E_3} \qquad \dfrac{\dfrac{x^{\alpha_4} \in \{x^{\alpha_4}\}}{x^{\alpha_4} \vdash x^{\alpha_5} \Rightarrow E_2}}{\vdash (\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6} \Rightarrow E_4}}{\vdash ((\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}(\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6})^{\alpha_7} \Rightarrow E_5}$$

Kitsendused:

$$\begin{aligned} E_5 \quad = \quad \{ & \\ & \alpha_2 = \alpha_5 \rightarrow \alpha_5, \\ & \\ & \alpha_2 = \alpha_7\} \end{aligned}$$

Type inference for $\lambda\!\rightarrow$ a'la Curry

$$\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1}}{\vdash (\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3} \Rightarrow E_3} \qquad \dfrac{\dfrac{x^{\alpha_4} \in \{x^{\alpha_4}\}}{x^{\alpha_4} \vdash x^{\alpha_5} \Rightarrow E_2}}{\vdash (\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6} \Rightarrow E_4}}{\vdash ((\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}(\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6})^{\alpha_7} \Rightarrow E_5}$$

Kitsendused:

$$E_5 = \{ \\ \alpha_7 = \alpha_5 \rightarrow \alpha_5 \\ \}$$

Tüübituletus $\lambda\!\rightarrow$ a'la Curry jaoks

$$\dfrac{}{\vdash (\lambda x^{\alpha_1}.(x^{\alpha_2} x^{\alpha_3})^{\alpha_4})^{\alpha_5}}$$

Tüübituletus $\lambda\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\overline{x^{\alpha_1} \vdash (x^{\alpha_2} x^{\alpha_3})^{\alpha_4}}}{\vdash (\lambda x^{\alpha_1}.(x^{\alpha_2} x^{\alpha_3})^{\alpha_4})^{\alpha_5}}$$

Tüübituletus $\lambda\!\to$ a'la Curry jaoks

$$\dfrac{\dfrac{}{x^{\alpha_1} \vdash x^{\alpha_2}} \qquad \dfrac{}{x^{\alpha_1} \vdash x^{\alpha_3}}}{\dfrac{x^{\alpha_1} \vdash (x^{\alpha_2} x^{\alpha_3})^{\alpha_4}}{\vdash (\lambda x^{\alpha_1}.(x^{\alpha_2} x^{\alpha_3})^{\alpha_4})^{\alpha_5}}}$$

Tüübituletus $\lambda\!\to$ a'la Curry jaoks

$$
\dfrac{
\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1}
\qquad
\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_3} \Rightarrow E_2}
}{
\dfrac{x^{\alpha_1} \vdash (x^{\alpha_2} x^{\alpha_3})^{\alpha_4}}{\vdash (\lambda x^{\alpha_1}.(x^{\alpha_2} x^{\alpha_3})^{\alpha_4})^{\alpha_5}}
}
$$

Kitsendused:

$$
\begin{array}{lcl}
E_1 & = & \{\alpha_1 = \alpha_2\} \\
E_2 & = & \{\alpha_1 = \alpha_3\}
\end{array}
$$

Tüübituletus $\lambda\!\to$ a'la Curry jaoks

$$\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1} \qquad \dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_3} \Rightarrow E_2}}{\dfrac{x^{\alpha_1} \vdash (x^{\alpha_2} x^{\alpha_3})^{\alpha_4} \Rightarrow E_3}{\vdash (\lambda x^{\alpha_1}.(x^{\alpha_2} x^{\alpha_3})^{\alpha_4})^{\alpha_5}}}$$

Kitsendused:

$$\begin{array}{lcl} E_1 & = & \{\alpha_1 = \alpha_2\} \\ E_2 & = & \{\alpha_1 = \alpha_3\} \\ E_3 & = & \{\alpha_2 = \alpha_3 \to \alpha_4\} \cup E_1 \cup E_2 \end{array}$$

Tüübituletus $\lambda\!\to$ a'la Curry jaoks

$$
\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1} \qquad \dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_3} \Rightarrow E_2}}{x^{\alpha_1} \vdash (x^{\alpha_2} x^{\alpha_3})^{\alpha_4} \Rightarrow E_3}}{\vdash (\lambda x^{\alpha_1}.(x^{\alpha_2} x^{\alpha_3})^{\alpha_4})^{\alpha_5} \Rightarrow E_4}
$$

Kitsendused:

$$
\begin{array}{lcl}
E_1 & = & \{\alpha_1 = \alpha_2\} \\
E_2 & = & \{\alpha_1 = \alpha_3\} \\
E_3 & = & \{\alpha_2 = \alpha_3 \to \alpha_4\} \cup E_1 \cup E_2 \\
E_4 & = & \{\alpha_5 = \alpha_1 \to \alpha_4\} \cup E_3
\end{array}
$$

Tüübituletus $\lambda\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1} \qquad \dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_3} \Rightarrow E_2}}{\dfrac{x^{\alpha_1} \vdash (x^{\alpha_2} x^{\alpha_3})^{\alpha_4} \Rightarrow E_3}{\vdash (\lambda x^{\alpha_1}.(x^{\alpha_2} x^{\alpha_3})^{\alpha_4})^{\alpha_5} \Rightarrow E_4}}$$

Kitsendused:

$$\begin{array}{rcl} E_4 & = & \{\alpha_1 = \alpha_2,\ \alpha_1 = \alpha_3, \\ & & \ \alpha_2 = \alpha_3 \rightarrow \alpha_4, \\ & & \ \alpha_5 = \alpha_1 \rightarrow \alpha_4 \\ & & \} \end{array}$$

Tüübituletus $\lambda\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1} \qquad \dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_3} \Rightarrow E_2}}{x^{\alpha_1} \vdash (x^{\alpha_2} x^{\alpha_3})^{\alpha_4} \Rightarrow E_3}}{\vdash (\lambda x^{\alpha_1}.(x^{\alpha_2} x^{\alpha_3})^{\alpha_4})^{\alpha_5} \Rightarrow E_4}$$

Kitsendused:

$$\begin{aligned} E_4 \quad = \quad \{ \quad & \alpha_2 = \alpha_3, \\ & \alpha_2 = \alpha_3 \rightarrow \alpha_4, \\ & \alpha_5 = \alpha_2 \rightarrow \alpha_4 \\ \} & \end{aligned}$$

Tüübituletus $\lambda\!\rightarrow$ a'la Curry jaoks

$$
\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1} \qquad \dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_3} \Rightarrow E_2}}{x^{\alpha_1} \vdash (x^{\alpha_2} x^{\alpha_3})^{\alpha_4} \Rightarrow E_3}}{\vdash (\lambda x^{\alpha_1}.(x^{\alpha_2} x^{\alpha_3})^{\alpha_4})^{\alpha_5} \Rightarrow E_4}
$$

Kitsendused:

$$
\begin{aligned}
E_4 \quad = \quad & \{ \\
& \quad \alpha_3 = \alpha_3 \rightarrow \alpha_4, \\
& \quad \alpha_5 = \alpha_3 \rightarrow \alpha_4 \\
& \}
\end{aligned}
$$

Tüübituletus $\lambda\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1} \qquad \dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_3} \Rightarrow E_2}}{x^{\alpha_1} \vdash (x^{\alpha_2} x^{\alpha_3})^{\alpha_4} \Rightarrow E_3}}{\vdash (\lambda x^{\alpha_1}.(x^{\alpha_2} x^{\alpha_3})^{\alpha_4})^{\alpha_5} \Rightarrow E_4}$$

Kitsendused:

$$\begin{array}{lll} E_4 & = & \{ \\ & & \quad \alpha_3 = \alpha_3 \to \alpha_4, \\ & & \quad \alpha_5 = \alpha_3 \to \alpha_4 \\ & & \} \end{array}$$

Error!

Tüübituletus

- Väga suure tüübiga term:

$$
\begin{array}{l}
\text{let } pair = \lambda xyz.z\,x\,y \text{ in} \\
\quad \text{let } x_1 = \lambda y.pair\,y\,y \text{ in} \\
\quad\quad \text{let } x_2 = \lambda y.x_1(x_1\,y) \text{ in} \\
\quad\quad\quad \text{let } x_3 = \lambda y.x_2(x_2\,y) \text{ in} \\
\quad\quad\quad\quad \text{let } x_4 = \lambda y.x_3(x_3\,y) \text{ in} \\
\quad\quad\quad\quad\quad \text{let } x_5 = \lambda y.x_4(x_4\,y) \text{ in} \\
\quad\quad\quad\quad\quad\quad x_5(\lambda y.y)
\end{array}
$$