Kava

* 12. loeng
    K: Interaktiivne teoreemitõestus

* 13. loeng
    K: Sõltuvad tüübid
    V: Tüübituletus

Interaktiivne teoreemitõestus

Loogikareegel:

$$\frac{A \quad B}{C}\ \text{nimi}$$

Loogikareegel Idrises:

```
nimi : a → b
       ------
    →    c
```

- Töötab, kuna Idrises on puhtad ja täielikud funktsioonid.
- Täielikuse jaoks võtmesõna `total` või kogu failile `%default total`
- Slaidide lugemine ei anna kogu infot edasi – interaktiivne tõestamine!

Näide: konjunktsioon

$$\frac{A \quad B}{A \wedge B}\ \mathrm{conj}_i$$

Idrises:

```
infixl 10 /\
data (/\) : Type → Type → Type where
    ConI : a → b
           ------
         → a /\ b
```

$$\frac{A \wedge B}{A}\ \text{conj}_{el} \qquad \frac{A \wedge B}{B}\ \text{conj}_{er}$$

Loogikareegel Idrises:

```
conEl : a /\ b
        ------
     →    a
conEl = ?conEl_v

conEr : a /\ b
        ------
     →    b
conEr = ?conEr_v
```

$$\frac{A \wedge B}{A}\ \mathrm{conj}_{el} \qquad \frac{A \wedge B}{B}\ \mathrm{conj}_{er}$$

koos tõestustega:

```
conEl : a /\ b
        ------
     →    a
conEl (ConI x y) = x

conEr : a /\ b
        ------
     →    b
conEr (ConI x y) = y
```

Võrdused + `rewrite`

```
-- defineeritud standardteegis
data (=) : a → b → Type where
  Refl :
         -----
         a = a


plusNull :   (n:Nat)
           ---------
         → n+0 = n

plusNull n = ?pl_null
```

Võrdused + `rewrite`

```
-- defineeritud standardteegis
data (=) : a → b → Type where
  Refl :
         -----
         a = a


plusNull :   (n:Nat)
           ---------
         → n+0 = n

plusNull 0 = Refl
plusNull (S k) = rewrite plusNull k in Refl
```

- Mitme `rewrite` puhul võib tekkida segadus. Kirjuta vahetulemused kommentaari!

Näide

```
plusAssoc : (m:Nat) → (n:Nat) → (q:Nat)
            -------------------------------
          →  m + (n + q) = (m + n) + q


plusAssoc 0 n q =
    {- plus n q = plus n q -}
    Refl

plusAssoc (S k) n q =
    {- S (plus k (plus n q)) = S (plus (plus k n) q) -}
    rewrite plusAssoc k n q  in  {- k + (n + q) = (k + n) + q -}
    {- S (plus (plus k n) q) = S (plus (plus k n) q) -}
    Refl
```

väär + absurd

```
-- defineeritud standardteegis
data Void : Type where
    -- konstruktoreid pole


absurd :  (p: Type) → Void
          ------------------
        →       p

absurd _ q impossible
```

väär + absurd

```
-- defineeritud standardteegis
data Void : Type where
    -- konstruktoreid pole


absurd :  (p: Type) → Void
          ------------------
        →       p

absurd _ q impossible
```

Kokkuvõtteks:

- Tüübiteooria on hea alus tõestussüsteemile!
- Idris sobib õppimiseks kuid pole piisavalt küps suurte tõestuste jaoks.
- Soovitame kasutada muid vahendeid. Näiteks Agda või Coq.

Kava

* 12. loeng
  K: Interaktiivne teoreemitõestus

* 13. loeng
  K: Sõltuvad tüübid
  V: Tüübituletus

Probleem

Tahame listi indekseerida funktsiooniga: `Nat` $\rightarrow$ `List a` $\rightarrow$ `a`

Probleem:

- List ei pruugi sisaldada `n`-ndat elementi!
- List võib olla täiesti tühi! Siis ei saa isegi valet elementi tagastada.

Võime teha nii: `Nat` $\rightarrow$ `List a` $\rightarrow$ `Maybe a`.

Töötab, kuid pole optimaalne, kui listis on piisavalt elemente.

Listi tüüp võiks sisaldada infot tema pikkuse kohta.

Pikkusega listid

- Tavalised listid (uue süntaksi abil)

```
data List : Type → Type where
    Nil : List a
    (::) : a → List a → List a
```

- Paneme pikkused külge

```
data Vec : Nat → Type → Type where
    Nil : Vec 0 a
    (::) : a → Vec n a → Vec (1+n) a

test5 : Vec 3 Int
test5 = [1,2,3]
```

- selekteerimine

```
nth : (n:Nat) → Vec (1+n+m) a → a
nth 0     (x :: xs) = x
nth (S k) (x :: xs) = nth k xs
```

- konkateneerimine

```
concatVec : Vec n a → Vec m a → Vec (n+m) a
concatVec []       ys = ys
concatVec (x :: y) ys = x :: concatVec y ys
```

- Vektorite map

```
Functor (Vec n) where
    -- map : (a → b) → Vec n a → Vec n b
    map f []      = []
    map f (x :: y) = f x :: map f y
```

Funktsiooni `nth n` tüüp `Vec (1+n+m) a → a` sõltub `n`-i väärtusest.

- `nth 0 : Vec (1+m) a → a`
- `nth 2 : Vec (3+m) a → a`

Sõltuvad tüübid võimaldavad arvutamist ja selle tõestamist koos teha.

Aga see ei tööta alati nii lihtsalt ...

- Ümberpööramine listidel

```
revList : List a → List a
revList [] = []
revList (x :: ys) = (revList ys) ++ [x]
```

- Ümberpööramine vektoritel

```
idVec : {n:Nat} → Vec (n + 1) a → Vec (1 + n) a
idVec {n = 0}     xs       = xs
idVec {n = (S k)} (x :: ys) = x :: idVec ys

revVec : {n:Nat} → Vec n a → Vec n a
revVec {n = 0  } []       = []
revVec {n = S k} (x :: ys) =
    idVec ((revVec ys) `concatVec` [x])
```

- Ilma `idVec`-ta tuleb veateade. Idris ei tea, et `k + (1 + 0)` on `S k` ehk `1+k`.

Tüübituletus

- Church'i-stiilis λ-arvutustes sisaldavad λ-termid piisavalt tüübiannotatsioone ning tüübikontroll ja tüübituletus on lihtne.
- Curry-stiilis süsteemides on termid ilma tüübiannotatsioonideta ja tüübid on termi korrektsuse predikaadid.
- Üldjuhul on Curry-stiilis tüübisüsteemides tüübituletus (ja ka tüübikontroll) lahendamatud ülesanded.
  - Teist-järku λ-arvutus.
- Kuid lihtsamate süsteemide jaoks on tüübituletus võimalik.
  - Lihtsalt tüübitud λ-arvutus.
  - Hindley-Milner'i polümorfism.

Curry-stiilis lihtsalt tüübitud λ-arvutus

- Tüübid (sama kui $\lambda\!\!\rightarrow$ a'la Church):

$$\begin{array}{rcll} \tau & ::= & \alpha & \text{tüübimuutuja} \\ & \mid & \tau_1 \to \tau_2 & \text{funktsioonitüüp} \end{array}$$

- Termid (sama kui puhtas λ-arvutuses):

$$\begin{array}{rcll} e & ::= & x & \text{muutuja} \\ & \mid & e_1\ e_2 & \text{aplikatsioon} \\ & \mid & \lambda x.\, e & \text{abstraktsioon} \end{array}$$

- Tüüpimisreeglid:

$$\frac{}{\Gamma, x : \tau \vdash x : \tau} \qquad \frac{\Gamma, x : \sigma \vdash e : \tau}{\Gamma \vdash \lambda x.\, e : \sigma \to \tau}$$

$$\frac{\Gamma \vdash e_1 : \sigma \to \tau \quad \Gamma \vdash e_2 : \sigma}{\Gamma \vdash e_1\ e_2 : \tau}$$

Curry-stiilis lihtsalt tüübitud λ-arvutus

- Tähistused:
  - $S, S', \ldots$ tüübisubstitutsioonid
  - $\tau \succ \tau' \iff \exists S\,[\tau' = S(\tau)]$;
  - $\Gamma \succ \Gamma' \iff \exists S\,[\Gamma' \supseteq S(\Gamma)]$.
- Definitsioon: (Γ, τ) on termi e printsipiaalne paar parajasti siis, kui
  (i) $\Gamma \vdash e : \tau$;
  (ii) $\Gamma' \vdash e : \tau' \iff \Gamma \succ \Gamma' \;\wedge\; \tau \succ \tau'$.
- Kinnise termi e printsipiaalses paaris $(\emptyset, \tau)$ olevat tüüpi τ nimetatakse printsipiaalseks tüübiks.
- Teoreem: Iga tüübitava termi e jaoks leidub printsipiaalne paar (Γ, τ). See paar on unikaalne tüübimuutujate ümbernimetamise täpsuseni.

Tüübituletus $\lambda\!\rightarrow$ a'la Curry jaoks

Tüübituletuse algoritm:

- Annoteerida iga alamavaldis ja muutuja siduvesinemine unikaalse tüübimuutujaga.
- Genereeri kitsenduste süsteem kasutades järgnevaid reegleid:

$$\frac{x^\alpha \in \Gamma}{\Gamma \vdash x^\beta \Rightarrow \{\alpha = \beta\}} \qquad \frac{\Gamma, x^\alpha \vdash e^\beta \Rightarrow E}{\Gamma \vdash (\lambda x^\alpha.\, e^\beta)^\gamma \Rightarrow \{\gamma = \alpha \rightarrow \beta\} \cup E}$$

$$\frac{\Gamma \vdash e_1^\alpha \Rightarrow E_1 \qquad \Gamma \vdash e_2^\beta \Rightarrow E_2}{\Gamma \vdash (e_1^\alpha\, e_2^\beta)^\gamma \Rightarrow \{\alpha = \beta \rightarrow \gamma\} \cup E_1 \cup E_2}$$

- Lahenda kitsenduste süsteem leides selle kõige üldisema unifitseerija.
  - Kui seda ei leidu $\Longrightarrow$ term ei ole tüübitav.

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5}z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2}}}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}}$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$
\dfrac{\dfrac{\dfrac{}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1}}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2}}}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}}
$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$
\begin{array}{c}
\hline
x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \\
\hline
x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1} \\
\hline
x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2} \\
\hline
\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}
\end{array}
$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash x^{\alpha_4}} \qquad \dfrac{}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1}}}{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2}}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1}}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2}}}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}}
$$

Tüübituletus $\lambda\!\to$ a'la Curry jaoks

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash x^{\alpha_4}} \qquad \dfrac{\dfrac{}{\Gamma \vdash y^{\alpha_5}} \quad \dfrac{}{\Gamma \vdash z^{\alpha_6}}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1}}}{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2}}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1}}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2}}}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}}
$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \qquad \dfrac{\dfrac{}{\Gamma \vdash y^{\alpha_5}} \qquad \dfrac{}{\Gamma \vdash z^{\alpha_6}}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1}}}{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2}}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1}}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2}}}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}}
$$

Kitsendused:

$$E_1 \quad = \quad \{\alpha_1 = \alpha_4\}$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \qquad \dfrac{\dfrac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \dfrac{}{\Gamma \vdash z^{\alpha_6}}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1}}}{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2}}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1}}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2}}}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}}
$$

Kitsendused:

$$
\begin{array}{lcl}
E_1 & = & \{\alpha_1 = \alpha_4\} \\
E_2 & = & \{\alpha_2 = \alpha_5\}
\end{array}
$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$
\dfrac{\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \quad \dfrac{\dfrac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \dfrac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1}}}{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2}}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1}}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2}}}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}}
$$

Kitsendused:

$$
\begin{array}{lcl}
E_1 & = & \{\alpha_1 = \alpha_4\} \\
E_2 & = & \{\alpha_2 = \alpha_5\} \\
E_3 & = & \{\alpha_3 = \alpha_6\}
\end{array}
$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$
\dfrac{\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \qquad \dfrac{\dfrac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \dfrac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4}}{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2}}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1}}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2}}}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}}
$$

Kitsendused:

$$
\begin{array}{lcl}
E_1 & = & \{\alpha_1 = \alpha_4\} \\
E_2 & = & \{\alpha_2 = \alpha_5\} \\
E_3 & = & \{\alpha_3 = \alpha_6\} \\
E_4 & = & \{\alpha_5 = \alpha_6 \rightarrow \beta_1\} \\
 & & \quad \cup\ E_2 \cup E_3
\end{array}
$$

Tüübituletus $\lambda\!\rightarrow$ a'la Curry jaoks

$$
\dfrac{\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \qquad \dfrac{\dfrac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \dfrac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4}}{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \Rightarrow E_5}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1}}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2}}}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}}
$$

Kitsendused:

$$
\begin{array}{lcl}
E_1 & = & \{\alpha_1 = \alpha_4\} \\
E_2 & = & \{\alpha_2 = \alpha_5\} \\
E_3 & = & \{\alpha_3 = \alpha_6\} \\
E_4 & = & \{\alpha_5 = \alpha_6 \rightarrow \beta_1\} \\
 & & \quad \cup\ E_2 \cup E_3
\end{array}
\qquad
E_5 \quad = \quad \{\alpha_4 = \beta_1 \rightarrow \beta_2\} \cup E_1 \cup E_4
$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \qquad \dfrac{\dfrac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \dfrac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4}}{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \Rightarrow E_5}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1} \Rightarrow E_6}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2}}}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}}
$$

Kitsendused:

$$
\begin{array}{lcl}
E_1 & = & \{\alpha_1 = \alpha_4\} \\
E_2 & = & \{\alpha_2 = \alpha_5\} \\
E_3 & = & \{\alpha_3 = \alpha_6\} \\
E_4 & = & \{\alpha_5 = \alpha_6 \rightarrow \beta_1\} \\
 & & \quad \cup\ E_2 \cup E_3
\end{array}
\qquad
\begin{array}{lcl}
E_5 & = & \{\alpha_4 = \beta_1 \rightarrow \beta_2\} \cup E_1 \cup E_4 \\
E_6 & = & \{\gamma_1 = \alpha_3 \rightarrow \beta_2\} \cup E_5
\end{array}
$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \qquad \dfrac{\dfrac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \dfrac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4}}{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \Rightarrow E_5}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1} \Rightarrow E_6}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2} \Rightarrow E_7}}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}}
$$

Kitsendused:

$$
\begin{array}{lcl}
E_1 & = & \{\alpha_1 = \alpha_4\} \\
E_2 & = & \{\alpha_2 = \alpha_5\} \\
E_3 & = & \{\alpha_3 = \alpha_6\} \\
E_4 & = & \{\alpha_5 = \alpha_6 \rightarrow \beta_1\} \\
 & & \quad \cup\ E_2 \cup E_3
\end{array}
\qquad
\begin{array}{lcl}
E_5 & = & \{\alpha_4 = \beta_1 \rightarrow \beta_2\} \cup E_1 \cup E_4 \\
E_6 & = & \{\gamma_1 = \alpha_3 \rightarrow \beta_2\} \cup E_5 \\
E_7 & = & \{\gamma_2 = \alpha_2 \rightarrow \gamma_1\} \cup E_6
\end{array}
$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \qquad \dfrac{\dfrac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \dfrac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4}}{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \Rightarrow E_5}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1} \Rightarrow E_6}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2} \Rightarrow E_7}}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3} \Rightarrow E_8}
$$

Kitsendused:

$$
\begin{array}{lcl}
E_1 & = & \{\alpha_1 = \alpha_4\} \\
E_2 & = & \{\alpha_2 = \alpha_5\} \\
E_3 & = & \{\alpha_3 = \alpha_6\} \\
E_4 & = & \{\alpha_5 = \alpha_6 \rightarrow \beta_1\} \\
 & & \quad \cup\ E_2 \cup E_3
\end{array}
\qquad
\begin{array}{lcl}
E_5 & = & \{\alpha_4 = \beta_1 \rightarrow \beta_2\} \cup E_1 \cup E_4 \\
E_6 & = & \{\gamma_1 = \alpha_3 \rightarrow \beta_2\} \cup E_5 \\
E_7 & = & \{\gamma_2 = \alpha_2 \rightarrow \gamma_1\} \cup E_6 \\
E_8 & = & \{\gamma_3 = \alpha_1 \rightarrow \gamma_2\} \cup E_7
\end{array}
$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \qquad \dfrac{\dfrac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \dfrac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4}}{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \Rightarrow E_5}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1} \Rightarrow E_6}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2} \Rightarrow E_7}}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3} \Rightarrow E_8}
$$

Kitsendused:

$$
\begin{aligned}
E_8 \quad = \quad & \{\alpha_1 = \alpha_4,\ \alpha_2 = \alpha_5,\ \alpha_3 = \alpha_6, \\
& \alpha_5 = \alpha_6 \rightarrow \beta_1,\ \alpha_4 = \beta_1 \rightarrow \beta_2, \\
& \gamma_1 = \alpha_3 \rightarrow \beta_2, \\
& \gamma_2 = \alpha_2 \rightarrow \gamma_1, \\
& \gamma_3 = \alpha_1 \rightarrow \gamma_2\}
\end{aligned}
$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \quad \dfrac{\dfrac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \dfrac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4}}{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \Rightarrow E_5}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1} \Rightarrow E_6}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2} \Rightarrow E_7}}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3} \Rightarrow E_8}
$$

Kitsendused:

$$
\begin{aligned}
E_8 \quad = \quad & \{\alpha_1 = \alpha_4,\ \alpha_2 = \alpha_5,\ \alpha_3 = \alpha_6, \\
& \alpha_5 = \alpha_6 \rightarrow \beta_1,\ \alpha_4 = \beta_1 \rightarrow \beta_2, \\
& \gamma_1 = \alpha_3 \rightarrow \beta_2, \\
& \gamma_2 = \alpha_2 \rightarrow \gamma_1, \\
& \gamma_3 = \alpha_4 \rightarrow \gamma_2\}
\end{aligned}
$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \qquad \dfrac{\dfrac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \dfrac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4}}{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \Rightarrow E_5}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1} \Rightarrow E_6}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2} \Rightarrow E_7}}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3} \Rightarrow E_8}
$$

Kitsendused:

$$
\begin{aligned}
E_8 \quad = \quad & \{\alpha_1 = \alpha_4,\ \alpha_2 = \alpha_5,\ \alpha_3 = \alpha_6, \\
& \alpha_5 = \alpha_6 \rightarrow \beta_1,\ \alpha_4 = \beta_1 \rightarrow \beta_2, \\
& \gamma_1 = \alpha_3 \rightarrow \beta_2, \\
& \gamma_2 = \alpha_5 \rightarrow \gamma_1, \\
& \gamma_3 = \alpha_4 \rightarrow \gamma_2\}
\end{aligned}
$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \qquad \dfrac{\dfrac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \dfrac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4}}{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \Rightarrow E_5}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1} \Rightarrow E_6}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2} \Rightarrow E_7}}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3} \Rightarrow E_8}
$$

Kitsendused:

$$
\begin{aligned}
E_8 \quad = \quad & \{\alpha_1 = \alpha_4,\ \alpha_2 = \alpha_5,\ \alpha_3 = \alpha_6, \\
& \alpha_5 = \alpha_6 \rightarrow \beta_1,\ \alpha_4 = \beta_1 \rightarrow \beta_2, \\
& \gamma_1 = \alpha_6 \rightarrow \beta_2, \\
& \gamma_2 = \alpha_5 \rightarrow \gamma_1, \\
& \gamma_3 = \alpha_4 \rightarrow \gamma_2\}
\end{aligned}
$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \qquad \dfrac{\dfrac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \dfrac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4}}{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \Rightarrow E_5}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1} \Rightarrow E_6}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2} \Rightarrow E_7}}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3} \Rightarrow E_8}
$$

Kitsendused:

$$
\begin{aligned}
E_8 \quad = \quad & \{\alpha_1 = \alpha_4,\ \alpha_2 = \alpha_5,\ \alpha_3 = \alpha_6, \\
& \alpha_5 = \alpha_6 \rightarrow \beta_1,\ \alpha_4 = \beta_1 \rightarrow \beta_2, \\
& \gamma_1 = \alpha_6 \rightarrow \beta_2, \\
& \gamma_2 = (\alpha_6 \rightarrow \beta_1) \rightarrow \gamma_1, \\
& \gamma_3 = \alpha_4 \rightarrow \gamma_2\}
\end{aligned}
$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \qquad \dfrac{\dfrac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \dfrac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4}}{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \Rightarrow E_5}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1} \Rightarrow E_6}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2} \Rightarrow E_7}}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3} \Rightarrow E_8}
$$

Kitsendused:

$$
\begin{aligned}
E_8 \quad = \quad & \{\alpha_1 = \alpha_4,\ \alpha_2 = \alpha_5,\ \alpha_3 = \alpha_6, \\
& \alpha_5 = \alpha_6 \rightarrow \beta_1,\ \alpha_4 = \beta_1 \rightarrow \beta_2, \\
& \gamma_1 = \alpha_6 \rightarrow \beta_2, \\
& \gamma_2 = (\alpha_6 \rightarrow \beta_1) \rightarrow \gamma_1, \\
& \gamma_3 = (\beta_1 \rightarrow \beta_2) \rightarrow \gamma_2\}
\end{aligned}
$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \qquad \dfrac{\dfrac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \dfrac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4}}{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \Rightarrow E_5}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1} \Rightarrow E_6}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2} \Rightarrow E_7}}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3} \Rightarrow E_8}
$$

Kitsendused:

$$
\begin{aligned}
E_8 \quad = \quad & \{\alpha_1 = \alpha_4,\ \alpha_2 = \alpha_5,\ \alpha_3 = \alpha_6, \\
& \alpha_5 = \alpha_6 \rightarrow \beta_1,\ \alpha_4 = \beta_1 \rightarrow \beta_2, \\
& \gamma_1 = \alpha_6 \rightarrow \beta_2, \\
& \gamma_2 = (\alpha_6 \rightarrow \beta_1) \rightarrow \alpha_6 \rightarrow \beta_2, \\
& \gamma_3 = (\beta_1 \rightarrow \beta_2) \rightarrow \gamma_2\}
\end{aligned}
$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$
\begin{array}{c}
\dfrac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \qquad \dfrac{\dfrac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \dfrac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4} \\
\hline
x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \Rightarrow E_5 \\
\hline
x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1} \Rightarrow E_6 \\
\hline
x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2} \Rightarrow E_7 \\
\hline
\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3} \Rightarrow E_8
\end{array}
$$

Kitsendused:

$$
\begin{aligned}
E_8 \quad = \quad & \{\alpha_1 = \alpha_4,\ \alpha_2 = \alpha_5,\ \alpha_3 = \alpha_6, \\
& \alpha_5 = \alpha_6 \rightarrow \beta_1,\ \alpha_4 = \beta_1 \rightarrow \beta_2, \\
& \gamma_1 = \alpha_6 \rightarrow \beta_2, \\
& \gamma_2 = (\alpha_6 \rightarrow \beta_1) \rightarrow \alpha_6 \rightarrow \beta_2, \\
& \gamma_3 = (\beta_1 \rightarrow \beta_2) \rightarrow (\alpha_6 \rightarrow \beta_1) \rightarrow \alpha_6 \rightarrow \beta_2\}
\end{aligned}
$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \quad \dfrac{\dfrac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \dfrac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4}}{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \Rightarrow E_5}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1} \Rightarrow E_6}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2} \Rightarrow E_7}}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3} \Rightarrow E_8}
$$

Tuletatud tüüp:

$$\gamma_3 \quad = \quad (\beta \rightarrow \gamma) \rightarrow (\alpha \rightarrow \beta) \rightarrow \alpha \rightarrow \gamma$$

Võrrandite lahendamine unifitseerimisega

- Võrrandeid saab lahendada lihtsustusreeglite korduva rakendamise kaudu.
- Kaks peamist reeglit:
  - asenda võrdus kujul $\tau_1 \to \tau_2 = \tau_3 \to \tau_4$ kahe võrdusega $\tau_1 = \tau_3$ ja $\tau_2 = \tau_4$;
  - olgu võrdus kujul $\alpha = \tau$. Kui $\alpha \in \mathrm{FV}(\tau)$ siis raporteeri viga, vastasel korral asenda kõigis võrdustes α asemel τ.
- Abireeglid:
  - eemalda reeglid kujul $\alpha = \alpha$, $\mathrm{Bool} = \mathrm{Bool}$, jne.;
  - asenda $\tau = \alpha$ võrdusega $\alpha = \tau$;
  - kui leidub võrdus $\tau_1 = \tau_2$, kus peatüübikonstruktorid on erinevad (näit. $\mathrm{Bool} = \alpha_1 \to \alpha_2$), siis raprorteeri viga.

Type inference for $\lambda\!\rightarrow$ a'la Curry

$$\vdash ((\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}(\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6})^{\alpha_7}$$

Type inference for $\lambda\!\rightarrow$ a'la Curry

$$\dfrac{\dfrac{}{\vdash (\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}} \qquad \dfrac{}{\vdash (\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6}}}{\vdash ((\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}(\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6})^{\alpha_7}}$$

Type inference for $\lambda\!\rightarrow$ a'la Curry

$$
\dfrac{\dfrac{\dfrac{}{x^{\alpha_1} \vdash x^{\alpha_2}}}{\vdash (\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}} \qquad \dfrac{\dfrac{}{x^{\alpha_4} \vdash x^{\alpha_5}}}{\vdash (\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6}}}{\vdash ((\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}(\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6})^{\alpha_7}}
$$

Type inference for $\lambda\!\rightarrow$ a'la Curry

$$
\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1}}{\vdash (\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}} \qquad \dfrac{\dfrac{x^{\alpha_4} \in \{x^{\alpha_4}\}}{x^{\alpha_4} \vdash x^{\alpha_5} \Rightarrow E_2}}{\vdash (\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6}}}{\vdash ((\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}(\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6})^{\alpha_7}}
$$

Kitsendused:

$$
\begin{array}{lcl}
E_1 & = & \{\alpha_1 = \alpha_2\} \\
E_2 & = & \{\alpha_4 = \alpha_5\}
\end{array}
$$

Type inference for $\lambda\!\rightarrow$ a'la Curry

$$\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1}}{\vdash (\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3} \Rightarrow E_3} \qquad \dfrac{\dfrac{x^{\alpha_4} \in \{x^{\alpha_4}\}}{x^{\alpha_4} \vdash x^{\alpha_5} \Rightarrow E_2}}{\vdash (\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6} \Rightarrow E_4}}{\vdash ((\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}(\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6})^{\alpha_7}}$$

Kitsendused:

$$\begin{array}{lcl} E_1 & = & \{\alpha_1 = \alpha_2\} \\ E_2 & = & \{\alpha_4 = \alpha_5\} \\ E_3 & = & \{\alpha_3 = \alpha_1 \rightarrow \alpha_2\} \cup E_1 \\ E_4 & = & \{\alpha_6 = \alpha_4 \rightarrow \alpha_5\} \cup E_2 \end{array}$$

Type inference for $\lambda\!\rightarrow$ a'la Curry

$$\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1}}{\vdash (\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3} \Rightarrow E_3} \qquad \dfrac{\dfrac{x^{\alpha_4} \in \{x^{\alpha_4}\}}{x^{\alpha_4} \vdash x^{\alpha_5} \Rightarrow E_2}}{\vdash (\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6} \Rightarrow E_4}}{\vdash ((\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}(\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6})^{\alpha_7} \Rightarrow E_5}$$

Kitsendused:

$$\begin{array}{lcl} E_1 & = & \{\alpha_1 = \alpha_2\} \\ E_2 & = & \{\alpha_4 = \alpha_5\} \\ E_3 & = & \{\alpha_3 = \alpha_1 \rightarrow \alpha_2\} \cup E_1 \\ E_4 & = & \{\alpha_6 = \alpha_4 \rightarrow \alpha_5\} \cup E_2 \\ E_5 & = & \{\alpha_3 = \alpha_6 \rightarrow \alpha_7\} \cup E_3 \cup E_4 \end{array}$$

Type inference for $\lambda\!\rightarrow$ a'la Curry

$$\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1}}{\vdash (\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3} \Rightarrow E_3} \qquad \dfrac{\dfrac{x^{\alpha_4} \in \{x^{\alpha_4}\}}{x^{\alpha_4} \vdash x^{\alpha_5} \Rightarrow E_2}}{\vdash (\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6} \Rightarrow E_4}}{\vdash ((\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}(\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6})^{\alpha_7} \Rightarrow E_5}$$

Kitsendused:

$$\begin{aligned} E_5 \quad = \quad & \{\alpha_1 = \alpha_2,\ \alpha_4 = \alpha_5, \\ & \alpha_3 = \alpha_1 \rightarrow \alpha_2, \\ & \alpha_6 = \alpha_4 \rightarrow \alpha_5, \\ & \alpha_3 = \alpha_6 \rightarrow \alpha_7\} \end{aligned}$$

Type inference for $\lambda\!\rightarrow$ a'la Curry

$$\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1}}{\vdash (\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3} \Rightarrow E_3} \qquad \dfrac{\dfrac{x^{\alpha_4} \in \{x^{\alpha_4}\}}{x^{\alpha_4} \vdash x^{\alpha_5} \Rightarrow E_2}}{\vdash (\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6} \Rightarrow E_4}}{\vdash ((\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}(\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6})^{\alpha_7} \Rightarrow E_5}$$

Kitsendused:

$$\begin{aligned} E_5 \quad = \quad \{ & \alpha_4 = \alpha_5, \\ & \alpha_3 = \alpha_2 \rightarrow \alpha_2, \\ & \alpha_6 = \alpha_4 \rightarrow \alpha_5, \\ & \alpha_3 = \alpha_6 \rightarrow \alpha_7 \} \end{aligned}$$

Type inference for $\lambda\!\rightarrow$ a'la Curry

$$\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1}}{\vdash (\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3} \Rightarrow E_3} \qquad \dfrac{\dfrac{x^{\alpha_4} \in \{x^{\alpha_4}\}}{x^{\alpha_4} \vdash x^{\alpha_5} \Rightarrow E_2}}{\vdash (\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6} \Rightarrow E_4}}{\vdash ((\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}(\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6})^{\alpha_7} \Rightarrow E_5}$$

Kitsendused:

$$\begin{aligned} E_5 \quad = \quad \{ & \\ & \alpha_3 = \alpha_2 \rightarrow \alpha_2, \\ & \alpha_6 = \alpha_5 \rightarrow \alpha_5, \\ & \alpha_3 = \alpha_6 \rightarrow \alpha_7 \} \end{aligned}$$

Type inference for $\lambda\!\rightarrow$ a'la Curry

$$
\dfrac{
\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1}}{\vdash (\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3} \Rightarrow E_3}
\qquad
\dfrac{\dfrac{x^{\alpha_4} \in \{x^{\alpha_4}\}}{x^{\alpha_4} \vdash x^{\alpha_5} \Rightarrow E_2}}{\vdash (\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6} \Rightarrow E_4}
}{\vdash ((\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}(\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6})^{\alpha_7} \Rightarrow E_5}
$$

Kitsendused:

$$
\begin{aligned}
E_5 \quad = \quad \{ & \\
& \alpha_6 = \alpha_5 \rightarrow \alpha_5, \\
& \alpha_2 \rightarrow \alpha_2 = \alpha_6 \rightarrow \alpha_7 \}
\end{aligned}
$$

Type inference for $\lambda\!\rightarrow$ a'la Curry

$$\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1}}{\vdash (\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3} \Rightarrow E_3} \qquad \dfrac{\dfrac{x^{\alpha_4} \in \{x^{\alpha_4}\}}{x^{\alpha_4} \vdash x^{\alpha_5} \Rightarrow E_2}}{\vdash (\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6} \Rightarrow E_4}}{\vdash ((\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}(\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6})^{\alpha_7} \Rightarrow E_5}$$

Kitsendused:

$$\begin{aligned} E_5 \quad = \quad \{ & \\ & \alpha_2 = \alpha_6, \\ & \alpha_6 = \alpha_5 \rightarrow \alpha_5, \\ & \alpha_2 = \alpha_7 \} \end{aligned}$$

Type inference for $\lambda\!\rightarrow$ a'la Curry

$$\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1}}{\vdash (\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3} \Rightarrow E_3} \qquad \dfrac{\dfrac{x^{\alpha_4} \in \{x^{\alpha_4}\}}{x^{\alpha_4} \vdash x^{\alpha_5} \Rightarrow E_2}}{\vdash (\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6} \Rightarrow E_4}}{\vdash ((\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}(\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6})^{\alpha_7} \Rightarrow E_5}$$

Kitsendused:

$$\begin{aligned} E_5 \quad = \quad \{ & \\ & \alpha_2 = \alpha_5 \rightarrow \alpha_5, \\ & \alpha_2 = \alpha_7 \} \end{aligned}$$

Type inference for $\lambda\!\rightarrow$ a'la Curry

$$\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1}}{\vdash (\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3} \Rightarrow E_3} \qquad \dfrac{\dfrac{x^{\alpha_4} \in \{x^{\alpha_4}\}}{x^{\alpha_4} \vdash x^{\alpha_5} \Rightarrow E_2}}{\vdash (\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6} \Rightarrow E_4}}{\vdash ((\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}(\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6})^{\alpha_7} \Rightarrow E_5}$$

Kitsendused:

$$E_5 = \{ \alpha_7 = \alpha_5 \rightarrow \alpha_5 \}$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$\frac{}{\vdash (\lambda x^{\alpha_1}.(x^{\alpha_2} x^{\alpha_3})^{\alpha_4})^{\alpha_5}}$$

Tüübituletus $\lambda\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{}{x^{\alpha_1} \vdash (x^{\alpha_2} x^{\alpha_3})^{\alpha_4}}}{\vdash (\lambda x^{\alpha_1}.(x^{\alpha_2} x^{\alpha_3})^{\alpha_4})^{\alpha_5}}$$

Tüübituletus $\lambda\!\rightarrow$ a'la Curry jaoks

$$
\dfrac{\dfrac{\dfrac{}{x^{\alpha_1} \vdash x^{\alpha_2}} \qquad \dfrac{}{x^{\alpha_1} \vdash x^{\alpha_3}}}{x^{\alpha_1} \vdash (x^{\alpha_2} x^{\alpha_3})^{\alpha_4}}}{\vdash (\lambda x^{\alpha_1}.(x^{\alpha_2} x^{\alpha_3})^{\alpha_4})^{\alpha_5}}
$$

Tüübituletus $\lambda\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1} \qquad \dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_3} \Rightarrow E_2}}{x^{\alpha_1} \vdash (x^{\alpha_2} x^{\alpha_3})^{\alpha_4}}}{\vdash (\lambda x^{\alpha_1}.(x^{\alpha_2} x^{\alpha_3})^{\alpha_4})^{\alpha_5}}$$

Kitsendused:

$$\begin{array}{lcl} E_1 & = & \{\alpha_1 = \alpha_2\} \\ E_2 & = & \{\alpha_1 = \alpha_3\} \end{array}$$

Tüübituletus $\lambda\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1} \qquad \dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_3} \Rightarrow E_2}}{x^{\alpha_1} \vdash (x^{\alpha_2} x^{\alpha_3})^{\alpha_4} \Rightarrow E_3}}{\vdash (\lambda x^{\alpha_1}.(x^{\alpha_2} x^{\alpha_3})^{\alpha_4})^{\alpha_5}}$$

Kitsendused:

$$\begin{array}{lcl} E_1 & = & \{\alpha_1 = \alpha_2\} \\ E_2 & = & \{\alpha_1 = \alpha_3\} \\ E_3 & = & \{\alpha_2 = \alpha_3 \rightarrow \alpha_4\} \cup E_1 \cup E_2 \end{array}$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1} \qquad \dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_3} \Rightarrow E_2}}{x^{\alpha_1} \vdash (x^{\alpha_2} x^{\alpha_3})^{\alpha_4} \Rightarrow E_3}}{\vdash (\lambda x^{\alpha_1}.(x^{\alpha_2} x^{\alpha_3})^{\alpha_4})^{\alpha_5} \Rightarrow E_4}$$

Kitsendused:

$$\begin{array}{lcl} E_1 & = & \{\alpha_1 = \alpha_2\} \\ E_2 & = & \{\alpha_1 = \alpha_3\} \\ E_3 & = & \{\alpha_2 = \alpha_3 \rightarrow \alpha_4\} \cup E_1 \cup E_2 \\ E_4 & = & \{\alpha_5 = \alpha_1 \rightarrow \alpha_4\} \cup E_3 \end{array}$$

Tüübituletus $\lambda\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1} \qquad \dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_3} \Rightarrow E_2}}{x^{\alpha_1} \vdash (x^{\alpha_2} x^{\alpha_3})^{\alpha_4} \Rightarrow E_3}}{\vdash (\lambda x^{\alpha_1}.(x^{\alpha_2} x^{\alpha_3})^{\alpha_4})^{\alpha_5} \Rightarrow E_4}$$

Kitsendused:

$$\begin{aligned} E_4 \quad = \quad & \{\alpha_1 = \alpha_2,\ \alpha_1 = \alpha_3, \\ & \ \alpha_2 = \alpha_3 \rightarrow \alpha_4, \\ & \ \alpha_5 = \alpha_1 \rightarrow \alpha_4 \\ & \} \end{aligned}$$

Tüübituletus $\lambda\!\rightarrow$ a'la Curry jaoks

$$
\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1} \qquad \dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_3} \Rightarrow E_2}}{x^{\alpha_1} \vdash (x^{\alpha_2} x^{\alpha_3})^{\alpha_4} \Rightarrow E_3}}{\vdash (\lambda x^{\alpha_1}.(x^{\alpha_2} x^{\alpha_3})^{\alpha_4})^{\alpha_5} \Rightarrow E_4}
$$

Kitsendused:

$$
\begin{array}{rcl}
E_4 & = & \{ \quad \alpha_2 = \alpha_3, \\
 & & \alpha_2 = \alpha_3 \rightarrow \alpha_4, \\
 & & \alpha_5 = \alpha_2 \rightarrow \alpha_4 \\
 & & \}
\end{array}
$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1} \qquad \dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_3} \Rightarrow E_2}}{x^{\alpha_1} \vdash (x^{\alpha_2} x^{\alpha_3})^{\alpha_4} \Rightarrow E_3}}{\vdash (\lambda x^{\alpha_1}.(x^{\alpha_2} x^{\alpha_3})^{\alpha_4})^{\alpha_5} \Rightarrow E_4}$$

Kitsendused:

$$\begin{array}{lcl} E_4 & = & \{ \\ & & \quad \alpha_3 = \alpha_3 \rightarrow \alpha_4, \\ & & \quad \alpha_5 = \alpha_3 \rightarrow \alpha_4 \\ & & \} \end{array}$$

Tüübituletus $\lambda\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1} \qquad \dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_3} \Rightarrow E_2}}{x^{\alpha_1} \vdash (x^{\alpha_2} x^{\alpha_3})^{\alpha_4} \Rightarrow E_3}}{\vdash (\lambda x^{\alpha_1}.(x^{\alpha_2} x^{\alpha_3})^{\alpha_4})^{\alpha_5} \Rightarrow E_4}$$

Kitsendused:

$$\begin{aligned} E_4 \quad = \quad & \{ \\ & \quad \alpha_3 = \alpha_3 \rightarrow \alpha_4, \\ & \quad \alpha_5 = \alpha_3 \rightarrow \alpha_4 \\ & \} \end{aligned}$$

Error!

Tüübituletus

- Väga suure tüübiga term:

$$
\begin{aligned}
&\text{let } pair = \lambda xyz.z\ x\ y \text{ in}\\
&\text{let } x_1 = \lambda y.pair\ y\ y \text{ in}\\
&\text{let } x_2 = \lambda y.x_1(x_1\ y) \text{ in}\\
&\text{let } x_3 = \lambda y.x_2(x_2\ y) \text{ in}\\
&\text{let } x_4 = \lambda y.x_3(x_3\ y) \text{ in}\\
&\text{let } x_5 = \lambda y.x_4(x_4\ y) \text{ in}\\
&x_5(\lambda y.y)
\end{aligned}
$$