

Interaktiivne teoreemitõestus

- Curry-Howardi vastavus ütleb: saame teha tõestusi prog. keelega

Loogikareegel:

$$\frac{A \quad B}{C} \text{ nimi}$$

Loogikareegel Idrises:

```
nimi : a → b
      -----
      → c
```

- Töötab, kuna Idrises on puhtad ja täielikud funktsioonid.
- Täielikuse jaoks võtmesõna `total` või kogu failile `%default total`
- Motivatsioon: saame tõestada midagi oma programmi kohta.

Näide: konjunktsioon

$$\frac{A \quad B}{A \wedge B} \text{conj}_i$$

Idrises:

```

infixl 10 /\
data (/\) : Type → Type → Type where
  ConI : a → b
        -----
        → a /\ b
  
```

$$\frac{A \wedge B}{A} \text{ conj}_{el}$$

$$\frac{A \wedge B}{B} \text{ conj}_{er}$$

Loogikareegel Idrises:

conEl : a ∧ b

→ a

conEl = ?conEl_v

conEr : a ∧ b

→ b

conEr = ?conEr_v

$$\frac{A \wedge B}{A} \text{ conj}_{el}$$

$$\frac{A \wedge B}{B} \text{ conj}_{er}$$

koos tõestustega:

$$\text{conEl} : a \wedge b$$

$$\begin{array}{c} \text{-----} \\ \rightarrow a \\ \text{conEl (ConI x y) = x} \end{array}$$

$$\text{conEr} : a \wedge b$$

$$\begin{array}{c} \text{-----} \\ \rightarrow b \\ \text{conEr (ConI x y) = y} \end{array}$$

Võrdused + rewrite

```
-- defineeritud standardteegis
data (≡) : a → b → Type where
  Refl :
    -----
    a = a
```

```
plusNull : (n:Nat)
           -----
           → n+0 = n
```

```
plusNull n = ?pl_null
```

Võrdused + rewrite

```

-- defineeritud standardteegis
data (≡) : a → b → Type where
  Refl :
    -----
    a = a

plusNull : (n:Nat)
           -----
           → n+0 = n

plusNull 0 = Refl
plusNull (S k) = rewrite plusNull k in Refl

```

- Mitme `rewrite` puhul võib tekkida segadus. Kirjuta vahetulemused kommentaari!

Näide

```
plusAssoc : (m:Nat) → (n:Nat) → (q:Nat)
-----
→ m + (n + q) = (m + n) + q
```

```
plusAssoc 0 n q =
  {- plus n q = plus n q -}
  Refl
```

```
plusAssoc (S k) n q =
  {- S (plus k (plus n q)) = S (plus (plus k n) q) -}
  rewrite plusAssoc k n q in {- k + (n + q) = (k + n) + q -}
  {- S (plus (plus k n) q) = S (plus (plus k n) q) -}
  Refl
```

väär + absurd

```
-- defineeritud standardteegis
data Void : Type where
  -- konstruktoreid pole
```

```
absurd : (p: Type) → Void
```

→ p

```
absurd _ q impossible
```


väär + absurd

```
-- defineeritud standardteegis
data Void : Type where
  -- konstruktoreid pole
```

```
absurd : (p: Type) → Void
         -----
         →      p
```

```
absurd _ q impossible
```

Kokkuvõtteks:

- Tüübiteooria on hea alus tõestussüsteemile!
- Idris sobib õppimiseks kuid pole piisavalt küps suurte tõestuste jaoks.
- Soovitame kasutada muid vahendeid. Näiteks Agda või Coq.

Tüübituletus

- **Church'i-stiilis** λ -arvutustes sisaldavad λ -terminid piisavalt tüübiannotatsioone ning tüübikontroll ja tüübituletus on lihtne.
- **Curry-stiilis** süsteemides on terminid ilma tüübiannotatsioonideta ja tüübid on termini korrektsuse predikaadid.
- Üldjuhul on Curry-stiilis tüübisüsteemides tüübituletus (ja ka tüübikontroll) lahendamatud ülesanded.
 - Teist-järku λ -arvutus.
- Kuid lihtsamate süsteemide jaoks on tüübituletus võimalik.
 - Lihtsalt tüübitud λ -arvutus.
 - Hindley-Milner'i polümorfism.

Curry-stiilis lihtsalt tüübitud λ -arvutus

- Tüübid (sama kui $\lambda \rightarrow$ a'la Church):

$$\begin{array}{l} \tau ::= \alpha \\ \quad | \tau_1 \rightarrow \tau_2 \end{array} \quad \begin{array}{l} \text{tüübimuutuja} \\ \text{funktsioonitüüp} \end{array}$$

- Termid (sama kui puhtas λ -arvutuses):

$$\begin{array}{l} e ::= x \\ \quad | e_1 e_2 \\ \quad | \lambda x. e \end{array} \quad \begin{array}{l} \text{muutuja} \\ \text{aplikatsioon} \\ \text{abstraktsioon} \end{array}$$

- Tüüpimisreeglid:

$$\frac{}{\Gamma, x : \tau \vdash x : \tau} \qquad \frac{\Gamma, x : \sigma \vdash e : \tau}{\Gamma \vdash \lambda x. e : \sigma \rightarrow \tau}$$

$$\frac{\Gamma \vdash e_1 : \sigma \rightarrow \tau \quad \Gamma \vdash e_2 : \sigma}{\Gamma \vdash e_1 e_2 : \tau}$$

Curry-stiilis lihtsalt tüübitud λ -arvutus

- Tähistused:
 - S, S', \dots tüübisubstitutsioonid
 - $\tau \succ \tau' \iff \exists S [\tau' = S(\tau)]$;
 - $\Gamma \succ \Gamma' \iff \exists S [\Gamma' \supseteq S(\Gamma)]$.
- **Definitsioon:** (Γ, τ) on termi e **printsipiaalne paar** parajasti siis, kui
 - $\Gamma \vdash e : \tau$;
 - $\Gamma' \vdash e : \tau' \iff \Gamma \succ \Gamma' \wedge \tau \succ \tau'$.
- Kinnise termi e printsipiaalses paaris (\emptyset, τ) olevat tüüpi τ nimetatakse **printsipiaalseks tüübiks**.
- **Teoreem:** Iga tüübitava termi e jaoks leidub printsipiaalne paar (Γ, τ) . See paar on unikaalne tüübimuutujate ümbernimetamise täpsuseni.

Tüübituletus $\lambda \rightarrow$ a'la Curry jaoks

Tüübituletuse algoritm:

- Annoteerida iga alamavaldis ja muutuja siduvestinimine unikaalse tüübimootujaga.
- Genereeri kitsenduste süsteem kasutades järgnevaid reegleid:

$$\frac{x^\alpha \in \Gamma}{\Gamma \vdash x^\beta \Rightarrow \{\alpha = \beta\}} \quad \frac{\Gamma, x^\alpha \vdash e^\beta \Rightarrow E}{\Gamma \vdash (\lambda x^\alpha. e^\beta)^\gamma \Rightarrow \{\gamma = \alpha \rightarrow \beta\} \cup E}$$

$$\frac{\Gamma \vdash e_1^\alpha \Rightarrow E_1 \quad \Gamma \vdash e_2^\beta \Rightarrow E_2}{\Gamma \vdash (e_1^\alpha e_2^\beta)^\gamma \Rightarrow \{\alpha = \beta \rightarrow \gamma\} \cup E_1 \cup E_2}$$

- Lahenda kitsenduste süsteem leides selle **kõige üldisema unifikseerija**.
 - Kui seda ei leidu \implies term ei ole tüübitav.

Tüübituletus $\lambda \rightarrow$ a'la Curry jaoks

$$\vdash (\lambda x^{\alpha_1} . (\lambda y^{\alpha_2} . (\lambda z^{\alpha_3} . (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}$$

Tüübituletus $\lambda \rightarrow$ a'la Curry jaoks

$$\frac{x^{\alpha_1} \vdash (\lambda y^{\alpha_2} . (\lambda z^{\alpha_3} . (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2}}{\vdash (\lambda x^{\alpha_1} . (\lambda y^{\alpha_2} . (\lambda z^{\alpha_3} . (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}}$$

Tüübituletus $\lambda \rightarrow$ a'la Curry jaoks

$$\frac{\frac{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3} . (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2} . (\lambda z^{\alpha_3} . (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2}}}{\vdash (\lambda x^{\alpha_1} . (\lambda y^{\alpha_2} . (\lambda z^{\alpha_3} . (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}}$$

Tüübituletus $\lambda \rightarrow a$ 'la Curry jaoks

$$\frac{\frac{\frac{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3} . (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2} . (\lambda z^{\alpha_3} . (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2}}}{\vdash (\lambda x^{\alpha_1} . (\lambda y^{\alpha_2} . (\lambda z^{\alpha_3} . (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}}$$

Tüübituletus $\lambda \rightarrow$ a'la Curry jaoks

$$\begin{array}{c}
 \frac{\Gamma \vdash x^{\alpha_4}}{\quad} \quad \frac{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1}}{\quad} \\
 \frac{\quad}{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2}} \\
 \frac{\quad}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1}} \\
 \frac{\quad}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}. (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2}} \\
 \frac{\quad}{\vdash (\lambda x^{\alpha_1}. (\lambda y^{\alpha_2}. (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}}
 \end{array}$$

Tüübituletus $\lambda \rightarrow$ a'la Curry jaoks

$$\begin{array}{c}
 \frac{\Gamma \vdash x^{\alpha_4} \quad \frac{\Gamma \vdash y^{\alpha_5} \quad \Gamma \vdash z^{\alpha_6}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1}}}{\Gamma \vdash (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2}} \\
 \frac{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1}} \\
 \frac{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}. (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2}}{\vdash (\lambda x^{\alpha_1}. (\lambda y^{\alpha_2}. (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}}
 \end{array}$$

Tüübituletus $\lambda \rightarrow$ a'la Curry jaoks

$$\begin{array}{c}
 \frac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \quad \frac{\Gamma \vdash y^{\alpha_5} \quad \Gamma \vdash z^{\alpha_6}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1}} \\
 \frac{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1}} \\
 \frac{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}. (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2}}{\vdash (\lambda x^{\alpha_1}. (\lambda y^{\alpha_2}. (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}}
 \end{array}$$

Kitsendused:

$$E_1 = \{\alpha_1 = \alpha_4\}$$

Tüübituletus $\lambda \rightarrow$ a'la Curry jaoks

$$\begin{array}{c}
 \frac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \quad \frac{\frac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \Gamma \vdash z^{\alpha_6}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1}}}{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2}} \\
 \frac{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}. (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2}} \\
 \frac{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}. (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2}}{\vdash (\lambda x^{\alpha_1}. (\lambda y^{\alpha_2}. (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}}
 \end{array}$$

Kitsendused:

$$E_1 = \{\alpha_1 = \alpha_4\}$$

$$E_2 = \{\alpha_2 = \alpha_5\}$$

Tüübituletus $\lambda \rightarrow$ a'la Curry jaoks

$$\begin{array}{c}
 \frac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \quad \frac{\frac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \frac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1}} \\
 \hline
 x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \\
 \hline
 x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1} \\
 \hline
 x^{\alpha_1} \vdash (\lambda y^{\alpha_2}. (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2} \\
 \hline
 \vdash (\lambda x^{\alpha_1}. (\lambda y^{\alpha_2}. (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}
 \end{array}$$

Kitsendused:

$$\begin{array}{lcl}
 E_1 & = & \{\alpha_1 = \alpha_4\} \\
 E_2 & = & \{\alpha_2 = \alpha_5\} \\
 E_3 & = & \{\alpha_3 = \alpha_6\}
 \end{array}$$

Tüübituletus $\lambda \rightarrow$ a'la Curry jaoks

$$\begin{array}{c}
 \frac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \quad \frac{\frac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \frac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4} \\
 \hline
 x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \\
 \hline
 x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1} \\
 \hline
 x^{\alpha_1} \vdash (\lambda y^{\alpha_2}. (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2} \\
 \hline
 \vdash (\lambda x^{\alpha_1}. (\lambda y^{\alpha_2}. (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}
 \end{array}$$

Kitsendused:

$$\begin{array}{lcl}
 E_1 & = & \{\alpha_1 = \alpha_4\} \\
 E_2 & = & \{\alpha_2 = \alpha_5\} \\
 E_3 & = & \{\alpha_3 = \alpha_6\} \\
 E_4 & = & \{\alpha_5 = \alpha_6 \rightarrow \beta_1\} \\
 & & \cup E_2 \cup E_3
 \end{array}$$

Tüübituletus $\lambda \rightarrow$ a'la Curry jaoks

$$\begin{array}{c}
 \frac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \quad \frac{\frac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \frac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4}}{\frac{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \Rightarrow E_5}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1}}} \\
 \frac{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}. (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2}} \\
 \frac{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}. (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2}}{\vdash (\lambda x^{\alpha_1}. (\lambda y^{\alpha_2}. (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}}
 \end{array}$$

Kitsendused:

$$\begin{array}{lcl}
 E_1 & = & \{\alpha_1 = \alpha_4\} \\
 E_2 & = & \{\alpha_2 = \alpha_5\} \\
 E_3 & = & \{\alpha_3 = \alpha_6\} \\
 E_4 & = & \{\alpha_5 = \alpha_6 \rightarrow \beta_1\} \\
 & & \cup E_2 \cup E_3 \\
 E_5 & = & \{\alpha_4 = \beta_1 \rightarrow \beta_2\} \cup E_1 \cup E_4
 \end{array}$$

Tüübituletus $\lambda \rightarrow$ a'la Curry jaoks

$$\begin{array}{c}
 \frac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \quad \frac{\frac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \frac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4} \\
 \frac{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \Rightarrow E_5}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1} \Rightarrow E_6} \\
 \frac{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}. (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2}}{\vdash (\lambda x^{\alpha_1}. (\lambda y^{\alpha_2}. (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}}
 \end{array}$$

Kitsendusud:

$$\begin{array}{lcl}
 E_1 & = & \{\alpha_1 = \alpha_4\} \\
 E_2 & = & \{\alpha_2 = \alpha_5\} \\
 E_3 & = & \{\alpha_3 = \alpha_6\} \\
 E_4 & = & \{\alpha_5 = \alpha_6 \rightarrow \beta_1\} \\
 & & \cup E_2 \cup E_3 \\
 E_5 & = & \{\alpha_4 = \beta_1 \rightarrow \beta_2\} \cup E_1 \cup E_4 \\
 E_6 & = & \{\gamma_1 = \alpha_3 \rightarrow \beta_2\} \cup E_5
 \end{array}$$

Tüübituletus $\lambda \rightarrow$ a'la Curry jaoks

$$\begin{array}{c}
 \frac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \quad \frac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \frac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3} \\
 \hline
 \Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4 \\
 \hline
 x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \Rightarrow E_5 \\
 \hline
 x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1} \Rightarrow E_6 \\
 \hline
 x^{\alpha_1} \vdash (\lambda y^{\alpha_2}. (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2} \Rightarrow E_7 \\
 \hline
 \vdash (\lambda x^{\alpha_1}. (\lambda y^{\alpha_2}. (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}
 \end{array}$$

Kitsendusud:

$$\begin{array}{ll}
 E_1 & = \{\alpha_1 = \alpha_4\} \\
 E_2 & = \{\alpha_2 = \alpha_5\} \\
 E_3 & = \{\alpha_3 = \alpha_6\} \\
 E_4 & = \{\alpha_5 = \alpha_6 \rightarrow \beta_1\} \\
 & \cup E_2 \cup E_3 \\
 E_5 & = \{\alpha_4 = \beta_1 \rightarrow \beta_2\} \cup E_1 \cup E_4 \\
 E_6 & = \{\gamma_1 = \alpha_3 \rightarrow \beta_2\} \cup E_5 \\
 E_7 & = \{\gamma_2 = \alpha_2 \rightarrow \gamma_1\} \cup E_6
 \end{array}$$

Tüübituletus $\lambda \rightarrow$ a'la Curry jaoks

$$\begin{array}{c}
 \frac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \quad \frac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \frac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3} \\
 \frac{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1 \quad \Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4}{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \Rightarrow E_5} \\
 \frac{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \Rightarrow E_5}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1} \Rightarrow E_6} \\
 \frac{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1} \Rightarrow E_6}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}. (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2} \Rightarrow E_7} \\
 \frac{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}. (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2} \Rightarrow E_7}{\vdash (\lambda x^{\alpha_1}. (\lambda y^{\alpha_2}. (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3} \Rightarrow E_8}
 \end{array}$$

Kitsendusud:

$$\begin{array}{ll}
 E_1 = \{\alpha_1 = \alpha_4\} & E_5 = \{\alpha_4 = \beta_1 \rightarrow \beta_2\} \cup E_1 \cup E_4 \\
 E_2 = \{\alpha_2 = \alpha_5\} & E_6 = \{\gamma_1 = \alpha_3 \rightarrow \beta_2\} \cup E_5 \\
 E_3 = \{\alpha_3 = \alpha_6\} & E_7 = \{\gamma_2 = \alpha_2 \rightarrow \gamma_1\} \cup E_6 \\
 E_4 = \{\alpha_5 = \alpha_6 \rightarrow \beta_1\} & E_8 = \{\gamma_3 = \alpha_1 \rightarrow \gamma_2\} \cup E_7 \\
 & \cup E_2 \cup E_3
 \end{array}$$

Tüübituletus $\lambda \rightarrow$ a'la Curry jaoks

$$\begin{array}{c}
 \frac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \quad \frac{\frac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \frac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4}}{\frac{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \Rightarrow E_5}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1} \Rightarrow E_6}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}. (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2} \Rightarrow E_7}}{\vdash (\lambda x^{\alpha_1}. (\lambda y^{\alpha_2}. (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3} \Rightarrow E_8}
 \end{array}$$

Kitsendused:

$$\begin{aligned}
 E_8 = \{ & \alpha_1 = \alpha_4, \alpha_2 = \alpha_5, \alpha_3 = \alpha_6, \\
 & \alpha_5 = \alpha_6 \rightarrow \beta_1, \alpha_4 = \beta_1 \rightarrow \beta_2, \\
 & \gamma_1 = \alpha_3 \rightarrow \beta_2, \\
 & \gamma_2 = \alpha_2 \rightarrow \gamma_1, \\
 & \gamma_3 = \alpha_1 \rightarrow \gamma_2 \}
 \end{aligned}$$

Tüübituletus $\lambda \rightarrow$ a'la Curry jaoks

$$\begin{array}{c}
 \frac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \quad \frac{\frac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \frac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4}}{\frac{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \Rightarrow E_5}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1} \Rightarrow E_6}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}. (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2} \Rightarrow E_7}}{\vdash (\lambda x^{\alpha_1}. (\lambda y^{\alpha_2}. (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3} \Rightarrow E_8}
 \end{array}$$

Kitsendused:

$$\begin{aligned}
 E_8 = \{ & \alpha_1 = \alpha_4, \alpha_2 = \alpha_5, \alpha_3 = \alpha_6, \\
 & \alpha_5 = \alpha_6 \rightarrow \beta_1, \alpha_4 = \beta_1 \rightarrow \beta_2, \\
 & \gamma_1 = \alpha_3 \rightarrow \beta_2, \\
 & \gamma_2 = \alpha_2 \rightarrow \gamma_1, \\
 & \gamma_3 = \alpha_4 \rightarrow \gamma_2 \}
 \end{aligned}$$

Tüübituletus $\lambda \rightarrow$ a'la Curry jaoks

$$\begin{array}{c}
 \frac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \quad \frac{\frac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \frac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4}}{\frac{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \Rightarrow E_5}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1} \Rightarrow E_6}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}. (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2} \Rightarrow E_7}}{\vdash (\lambda x^{\alpha_1}. (\lambda y^{\alpha_2}. (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3} \Rightarrow E_8}
 \end{array}$$

Kitsendused:

$$\begin{aligned}
 E_8 = \{ & \alpha_1 = \alpha_4, \alpha_2 = \alpha_5, \alpha_3 = \alpha_6, \\
 & \alpha_5 = \alpha_6 \rightarrow \beta_1, \alpha_4 = \beta_1 \rightarrow \beta_2, \\
 & \gamma_1 = \alpha_3 \rightarrow \beta_2, \\
 & \gamma_2 = \alpha_5 \rightarrow \gamma_1, \\
 & \gamma_3 = \alpha_4 \rightarrow \gamma_2 \}
 \end{aligned}$$

Tüübituletus $\lambda \rightarrow$ a'la Curry jaoks

$$\begin{array}{c}
 \frac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \quad \frac{\frac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \frac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4}}{\frac{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \Rightarrow E_5}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1} \Rightarrow E_6}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}. (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2} \Rightarrow E_7}}{\vdash (\lambda x^{\alpha_1}. (\lambda y^{\alpha_2}. (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3} \Rightarrow E_8}
 \end{array}$$

Kitsendused:

$$\begin{aligned}
 E_8 = \{ & \alpha_1 = \alpha_4, \alpha_2 = \alpha_5, \alpha_3 = \alpha_6, \\
 & \alpha_5 = \alpha_6 \rightarrow \beta_1, \alpha_4 = \beta_1 \rightarrow \beta_2, \\
 & \gamma_1 = \alpha_6 \rightarrow \beta_2, \\
 & \gamma_2 = \alpha_5 \rightarrow \gamma_1, \\
 & \gamma_3 = \alpha_4 \rightarrow \gamma_2 \}
 \end{aligned}$$

Tüübituletus $\lambda \rightarrow$ a'la Curry jaoks

$$\begin{array}{c}
 \frac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \quad \frac{\frac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \frac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4}}{\frac{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \Rightarrow E_5}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1} \Rightarrow E_6}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}. (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2} \Rightarrow E_7}}{\vdash (\lambda x^{\alpha_1}. (\lambda y^{\alpha_2}. (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3} \Rightarrow E_8}
 \end{array}$$

Kitsendused:

$$\begin{aligned}
 E_8 = \{ & \alpha_1 = \alpha_4, \alpha_2 = \alpha_5, \alpha_3 = \alpha_6, \\
 & \alpha_5 = \alpha_6 \rightarrow \beta_1, \alpha_4 = \beta_1 \rightarrow \beta_2, \\
 & \gamma_1 = \alpha_6 \rightarrow \beta_2, \\
 & \gamma_2 = (\alpha_6 \rightarrow \beta_1) \rightarrow \gamma_1, \\
 & \gamma_3 = \alpha_4 \rightarrow \gamma_2 \}
 \end{aligned}$$

Tüübituletus $\lambda \rightarrow$ a'la Curry jaoks

$$\begin{array}{c}
 \frac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \quad \frac{\frac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \frac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4}}{\frac{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \Rightarrow E_5}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2}) \gamma_1 \Rightarrow E_6}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}. (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2}) \gamma_1) \gamma_2 \Rightarrow E_7}}{\vdash (\lambda x^{\alpha_1}. (\lambda y^{\alpha_2}. (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2}) \gamma_1) \gamma_2) \gamma_3 \Rightarrow E_8}
 \end{array}$$

Kitsendused:

$$\begin{aligned}
 E_8 &= \{ \alpha_1 = \alpha_4, \alpha_2 = \alpha_5, \alpha_3 = \alpha_6, \\
 &\quad \alpha_5 = \alpha_6 \rightarrow \beta_1, \alpha_4 = \beta_1 \rightarrow \beta_2, \\
 &\quad \gamma_1 = \alpha_6 \rightarrow \beta_2, \\
 &\quad \gamma_2 = (\alpha_6 \rightarrow \beta_1) \rightarrow \gamma_1, \\
 &\quad \gamma_3 = (\beta_1 \rightarrow \beta_2) \rightarrow \gamma_2 \}
 \end{aligned}$$

Tüübituletus $\lambda \rightarrow$ a'la Curry jaoks

$$\begin{array}{c}
 \frac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \quad \frac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \frac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3} \\
 \frac{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1 \quad \Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4}{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \Rightarrow E_5} \\
 \frac{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1} \Rightarrow E_6}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}. (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2} \Rightarrow E_7} \\
 \frac{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}. (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2} \Rightarrow E_7}{\vdash (\lambda x^{\alpha_1}. (\lambda y^{\alpha_2}. (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3} \Rightarrow E_8}
 \end{array}$$

Kitsendused:

$$\begin{aligned}
 E_8 = & \{ \alpha_1 = \alpha_4, \alpha_2 = \alpha_5, \alpha_3 = \alpha_6, \\
 & \alpha_5 = \alpha_6 \rightarrow \beta_1, \alpha_4 = \beta_1 \rightarrow \beta_2, \\
 & \gamma_1 = \alpha_6 \rightarrow \beta_2, \\
 & \gamma_2 = (\alpha_6 \rightarrow \beta_1) \rightarrow \alpha_6 \rightarrow \beta_2, \\
 & \gamma_3 = (\beta_1 \rightarrow \beta_2) \rightarrow \gamma_2 \}
 \end{aligned}$$

Tüübituletus $\lambda \rightarrow$ a'la Curry jaoks

$$\begin{array}{c}
\frac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \quad \frac{\frac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \frac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4} \\
\frac{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \Rightarrow E_5}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1} \Rightarrow E_6} \\
\frac{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}. (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2} \Rightarrow E_7}{\vdash (\lambda x^{\alpha_1}. (\lambda y^{\alpha_2}. (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3} \Rightarrow E_8}
\end{array}$$

Kitsendused:

$$\begin{aligned}
E_8 = & \{ \alpha_1 = \alpha_4, \alpha_2 = \alpha_5, \alpha_3 = \alpha_6, \\
& \alpha_5 = \alpha_6 \rightarrow \beta_1, \alpha_4 = \beta_1 \rightarrow \beta_2, \\
& \gamma_1 = \alpha_6 \rightarrow \beta_2, \\
& \gamma_2 = (\alpha_6 \rightarrow \beta_1) \rightarrow \alpha_6 \rightarrow \beta_2, \\
& \gamma_3 = (\beta_1 \rightarrow \beta_2) \rightarrow (\alpha_6 \rightarrow \beta_1) \rightarrow \alpha_6 \rightarrow \beta_2 \}
\end{aligned}$$

Tüübituletus $\lambda \rightarrow$ a'la Curry jaoks

$$\begin{array}{c}
 \frac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \quad \frac{\frac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \frac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4}}{\frac{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \Rightarrow E_5}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1} \Rightarrow E_6}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}. (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2} \Rightarrow E_7}}{\vdash (\lambda x^{\alpha_1}. (\lambda y^{\alpha_2}. (\lambda z^{\alpha_3}. (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3} \Rightarrow E_8}
 \end{array}$$

Tuletatud tüüp:

$$\gamma_3 = (\beta \rightarrow \gamma) \rightarrow (\alpha \rightarrow \beta) \rightarrow \alpha \rightarrow \gamma$$