Tugev normaliseerimine

- Tugev normaliseerimine:
  Lihtsalt tüübitud λ-arvutuses $\lambda{\to}$ on iga β-reduktsiooni jada lõplik.
- Mõned lihtsad järeldused:
  - Termide võrdsuse küsimus on lahenduv.
  - Püsipunktikombinaatorid ei ole defineeritavad.

Laiendus: tõeväärtused

- Tüübid: $\tau ::= \ldots \mid \mathsf{Bool}$
- Termid: $e ::= \ldots \mid \mathsf{true} \mid \mathsf{false} \mid \mathsf{if}\ e_0\ \mathsf{then}\ e_1\ \mathsf{else}\ e_2$
- Väärtused: $v ::= \ldots \mid \mathsf{true} \mid \mathsf{false}$
- Tüüpimisreeglid:

$$\frac{}{\Gamma \vdash \mathsf{true} : \mathsf{Bool}} \qquad \frac{}{\Gamma \vdash \mathsf{false} : \mathsf{Bool}}$$

$$\frac{\Gamma \vdash e_0 : \mathsf{Bool} \quad \Gamma \vdash e_1 : \tau \quad \Gamma \vdash e_2 : \tau}{\Gamma \vdash \mathsf{if}\ e_0\ \mathsf{then}\ e_1\ \mathsf{else}\ e_2 : \tau}$$

- Väärtustamisreeglid:

$$\begin{array}{lcl} \mathsf{if\ true\ then}\ e_1\ \mathsf{else}\ e_2 & \to & e_1 \\ \mathsf{if\ false\ then}\ e_1\ \mathsf{else}\ e_2 & \to & e_2 \end{array}$$

Laiendus: ühiktüüp

- Tüübid: $\tau ::= \ldots \mid \mathrm{Unit}$
- Termid: $e ::= \ldots \mid ()$
- Väärtused: $v ::= \ldots \mid ()$
- Tüüpimisreeglid:

$$\frac{}{\Gamma \vdash () : \mathrm{Unit}}$$

- Väärtustamisreegleid ei ole!

Laiendus: paarid

- Tüübid: $\tau ::= \ldots \mid \tau_1 \times \tau_2$
- Termid: $e ::= \ldots \mid (e_1, e_2) \mid \mathtt{fst} \mid \mathtt{snd}$
- Väärtused: $v ::= \ldots \mid (v_1, v_2)$
- Tüüpimisreeglid:

$$\frac{\Gamma \vdash e_1 : \tau_1 \quad \Gamma \vdash e_2 : \tau_2}{\Gamma \vdash (e_1, e_2) : \tau_1 \times \tau_2}$$

$$\frac{\Gamma \vdash e : \tau_1 \times \tau_2}{\Gamma \vdash \mathtt{fst}\, e : \tau_1} \qquad \frac{\Gamma \vdash e : \tau_1 \times \tau_2}{\Gamma \vdash \mathtt{snd}\, e : \tau_2}$$

- Väärtustamisreeglid:

$$\begin{array}{lcl} \mathtt{fst}\,(e_1, e_2) & \to & e_1 \\ \mathtt{snd}\,(e_1, e_2) & \to & e_2 \end{array}$$

Laiendus: summatüüp

- Tüübid: $\tau ::= \ldots \mid \tau_1 + \tau_2$
- Termid: $e ::= \ldots \mid \mathtt{inl} \mid \mathtt{inr} \mid \mathsf{case}\,(e_0;\ x_1.e_1;\ x_2.e_2)$
- Väärtused: $v ::= \ldots \mid \mathtt{inl}\ v_1 \mid \mathtt{inl}\ v_2$
- Tüüpimisreeglid:

$$\frac{\Gamma \vdash e : \tau_1}{\Gamma \vdash \mathtt{inl}\ e : \tau_1 + \tau_2} \qquad \frac{\Gamma \vdash e : \tau_2}{\Gamma \vdash \mathtt{inr}\ e : \tau_1 + \tau_2}$$

$$\frac{\Gamma \vdash e_0 : \tau_1 + \tau_2 \quad \Gamma,\ x_1 : \tau_1 \vdash e_1 : \sigma \quad \Gamma,\ x_2 : \tau_2 \vdash e_2 : \sigma}{\Gamma \vdash \mathsf{case}\,(e_0;\ x_1.e_1;\ x_2.e_2) : \sigma}$$

- Evaluation rules:

$$\begin{aligned} \mathsf{case}\,(\mathtt{inl}\ e_0;\ x_1.e_1;\ x_2.e_2) &\to e_1[x_1 \mapsto e_0] \\ \mathsf{case}\,(\mathtt{inr}\ e_0;\ x_1.e_1;\ x_2.e_2) &\to e_2[x_2 \mapsto e_0] \end{aligned}$$

Tüübituletus

- Church'i-stiilis λ-arvutustes sisaldavad λ-termid piisavalt tüübiannotatsioone ning tüübikontroll ja tüübituletus on lihtne.
- Curry-stiilis süsteemides on termid ilma tüübiannotatsioonideta ja tüübid on termi korrektsuse predikaadid.
- Üldjuhul on Curry-stiilis tüübisüsteemides tüübituletus (ja ka tüübikontroll) lahendamatud ülesanded.
  - Teist-järku λ-arvutus.
- Kuid lihtsamate süsteemide jaoks on tüübituletus võimalik.
  - Lihtsalt tüübitud λ-arvutus.
  - Hindley-Milner'i polümorfism.

Curry-stiilis lihtsalt tüübitud λ-arvutus

- Tüübid (sama kui $\lambda\!\!\rightarrow$ a'la Church):

$$\begin{array}{rcll} \tau & ::= & \alpha & \text{tüübimuutuja} \\ & | & \tau_1 \to \tau_2 & \text{funktsioonitüüp} \end{array}$$

- Termid (sama kui puhtas λ-arvutuses):

$$\begin{array}{rcll} e & ::= & x & \text{muutuja} \\ & | & e_1\ e_2 & \text{aplikatsioon} \\ & | & \lambda x.\, e & \text{abstraktsioon} \end{array}$$

- Tüüpimisreeglid:

$$\frac{}{\Gamma, x : \tau \vdash x : \tau} \qquad \frac{\Gamma, x : \sigma \vdash e : \tau}{\Gamma \vdash \lambda x.\, e : \sigma \to \tau}$$

$$\frac{\Gamma \vdash e_1 : \sigma \to \tau \quad \Gamma \vdash e_2 : \sigma}{\Gamma \vdash e_1\ e_2 : \tau}$$

Curry-stiilis lihtsalt tüübitud λ-arvutus

- Tähistused:
  - $S, S', \ldots$ tüübisubstitutsioonid
  - $\tau \succ \tau' \iff \exists S\,[\tau' = S(\tau)]$;
  - $\Gamma \succ \Gamma' \iff \exists S\,[\Gamma' \supseteq S(\Gamma)]$.
- Definitsioon: (Γ, τ) on termi e printsipiaalne paar parajasti siis, kui
  (i) $\Gamma \vdash e : \tau$;
  (ii) $\Gamma' \vdash e : \tau' \iff \Gamma \succ \Gamma' \ \wedge \ \tau \succ \tau'$.
- Kinnise termi e printsipiaalses paaris $(\emptyset, \tau)$ olevat tüüpi τ nimetatakse printsipiaalseks tüübiks.
- Teoreem: Iga tüübitava termi e jaoks leidub printsipiaalne paar (Γ, τ). See paar on unikaalne tüübimuutujate ümbernimetamise täpsuseni.

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

Tüübituletuse algoritm:

- Annoteerida iga alamavaldis ja muutuja siduvesinemine unikaalse tüübimuutujaga.
- Genereeri kitsenduste süsteem kasutades järgnevaid reegleid:

$$\frac{x^\alpha \in \Gamma}{\Gamma \vdash x^\beta \Rightarrow \{\alpha = \beta\}} \qquad \frac{\Gamma, x^\alpha \vdash e^\beta \Rightarrow E}{\Gamma \vdash (\lambda x^\alpha.\, e^\beta)^\gamma \Rightarrow \{\gamma = \alpha \to \beta\} \cup E}$$

$$\frac{\Gamma \vdash e_1^\alpha \Rightarrow E_1 \qquad \Gamma \vdash e_2^\beta \Rightarrow E_2}{\Gamma \vdash (e_1^\alpha\ e_2^\beta)^\gamma \Rightarrow \{\alpha = \beta \to \gamma\} \cup E_1 \cup E_2}$$

- Lahenda kitsenduste süsteem leides selle kõige üldisema unifitseerija.
  - Kui seda ei leidu $\Longrightarrow$ term ei ole tüübitav.

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5}\, z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2}}}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}}$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1}}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2}}}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}}$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{\dfrac{}{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2}}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1}}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2}}}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}}$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash x^{\alpha_4}} \quad \dfrac{}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1}}}{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2}}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1}}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2}}}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}}$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash x^{\alpha_4}} \quad \dfrac{\dfrac{}{\Gamma \vdash y^{\alpha_5}} \quad \dfrac{}{\Gamma \vdash z^{\alpha_6}}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1}}}{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2}}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1}}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2}}}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4} (y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}}$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \quad \dfrac{\overline{\Gamma \vdash y^{\alpha_5}} \quad \overline{\Gamma \vdash z^{\alpha_6}}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1}}}{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2}}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1}}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2}}}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}}$$

Kitsendused:

$$E_1 \quad = \quad \{\alpha_1 = \alpha_4\}$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \quad \dfrac{\dfrac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \dfrac{}{\Gamma \vdash z^{\alpha_6}}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1}}}{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2}}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1}}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2}}}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}}$$

Kitsendused:

$$\begin{aligned} E_1 &= \{\alpha_1 = \alpha_4\} \\ E_2 &= \{\alpha_2 = \alpha_5\} \end{aligned}$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \quad \dfrac{\dfrac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \dfrac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1}}}{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2}}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1}}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2}}}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}}$$

Kitsendused:

$$\begin{array}{lcl} E_1 & = & \{\alpha_1 = \alpha_4\} \\ E_2 & = & \{\alpha_2 = \alpha_5\} \\ E_3 & = & \{\alpha_3 = \alpha_6\} \end{array}$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \quad \dfrac{\dfrac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \dfrac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4}}{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2}}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1}}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2}}}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}}$$

Kitsendused:

$$\begin{array}{lcl} E_1 & = & \{\alpha_1 = \alpha_4\} \\ E_2 & = & \{\alpha_2 = \alpha_5\} \\ E_3 & = & \{\alpha_3 = \alpha_6\} \\ E_4 & = & \{\alpha_5 = \alpha_6 \rightarrow \beta_1\} \\ & & \quad \cup\, E_2 \cup E_3 \end{array}$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \quad \dfrac{\dfrac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \dfrac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4}}{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \Rightarrow E_5}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1}}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2}}}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}}$$

Kitsendused:

$$\begin{aligned} E_1 &= \{\alpha_1 = \alpha_4\} \\ E_2 &= \{\alpha_2 = \alpha_5\} \\ E_3 &= \{\alpha_3 = \alpha_6\} \\ E_4 &= \{\alpha_5 = \alpha_6 \rightarrow \beta_1\} \\ &\quad \cup E_2 \cup E_3 \end{aligned} \qquad E_5 = \{\alpha_4 = \beta_1 \rightarrow \beta_2\} \cup E_1 \cup E_4$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \quad \dfrac{\dfrac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \dfrac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4}}{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \Rightarrow E_5}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1} \Rightarrow E_6}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2}}}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}}$$

Kitsendused:

$$\begin{aligned}
E_1 &= \{\alpha_1 = \alpha_4\} \\
E_2 &= \{\alpha_2 = \alpha_5\} \\
E_3 &= \{\alpha_3 = \alpha_6\} \\
E_4 &= \{\alpha_5 = \alpha_6 \rightarrow \beta_1\} \\
&\quad \cup E_2 \cup E_3
\end{aligned}$$

$$\begin{aligned}
E_5 &= \{\alpha_4 = \beta_1 \rightarrow \beta_2\} \cup E_1 \cup E_4 \\
E_6 &= \{\gamma_1 = \alpha_3 \rightarrow \beta_2\} \cup E_5
\end{aligned}$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{x^{\alpha_1} \in \Gamma}}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \quad \dfrac{\dfrac{\dfrac{}{y^{\alpha_2} \in \Gamma}}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \dfrac{\dfrac{}{z^{\alpha_3} \in \Gamma}}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4}}{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \Rightarrow E_5}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1} \Rightarrow E_6}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2} \Rightarrow E_7}}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3}}$$

Kitsendused:

$$\begin{array}{lcl} E_1 & = & \{\alpha_1 = \alpha_4\} \\ E_2 & = & \{\alpha_2 = \alpha_5\} \\ E_3 & = & \{\alpha_3 = \alpha_6\} \\ E_4 & = & \{\alpha_5 = \alpha_6 \rightarrow \beta_1\} \\ & & \cup\ E_2 \cup E_3 \end{array} \qquad \begin{array}{lcl} E_5 & = & \{\alpha_4 = \beta_1 \rightarrow \beta_2\} \cup E_1 \cup E_4 \\ E_6 & = & \{\gamma_1 = \alpha_3 \rightarrow \beta_2\} \cup E_5 \\ E_7 & = & \{\gamma_2 = \alpha_2 \rightarrow \gamma_1\} \cup E_6 \end{array}$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \quad \dfrac{\dfrac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \dfrac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4}}{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \Rightarrow E_5}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1} \Rightarrow E_6}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2} \Rightarrow E_7}}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3} \Rightarrow E_8}$$

Kitsendused:

$$\begin{array}{lcl} E_1 & = & \{\alpha_1 = \alpha_4\} \\ E_2 & = & \{\alpha_2 = \alpha_5\} \\ E_3 & = & \{\alpha_3 = \alpha_6\} \\ E_4 & = & \{\alpha_5 = \alpha_6 \rightarrow \beta_1\} \\ & & \cup\ E_2 \cup E_3 \end{array} \qquad \begin{array}{lcl} E_5 & = & \{\alpha_4 = \beta_1 \rightarrow \beta_2\} \cup E_1 \cup E_4 \\ E_6 & = & \{\gamma_1 = \alpha_3 \rightarrow \beta_2\} \cup E_5 \\ E_7 & = & \{\gamma_2 = \alpha_2 \rightarrow \gamma_1\} \cup E_6 \\ E_8 & = & \{\gamma_3 = \alpha_1 \rightarrow \gamma_2\} \cup E_7 \end{array}$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \quad \dfrac{\dfrac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \dfrac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4}}{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \Rightarrow E_5}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1} \Rightarrow E_6}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2} \Rightarrow E_7}}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3} \Rightarrow E_8}$$

Kitsendused:

$$\begin{aligned} E_8 \quad = \quad & \{\alpha_1 = \alpha_4,\ \alpha_2 = \alpha_5,\ \alpha_3 = \alpha_6, \\ & \alpha_5 = \alpha_6 \rightarrow \beta_1,\ \alpha_4 = \beta_1 \rightarrow \beta_2, \\ & \gamma_1 = \alpha_3 \rightarrow \beta_2, \\ & \gamma_2 = \alpha_2 \rightarrow \gamma_1, \\ & \gamma_3 = \alpha_1 \rightarrow \gamma_2\} \end{aligned}$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{x^{\alpha_1} \in \Gamma}}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \quad \dfrac{\dfrac{}{y^{\alpha_2} \in \Gamma}}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \dfrac{\dfrac{}{z^{\alpha_3} \in \Gamma}}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4}}{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \Rightarrow E_5}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1} \Rightarrow E_6}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2} \Rightarrow E_7}}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3} \Rightarrow E_8}$$

Kitsendused:

$$\begin{aligned} E_8 \quad = \quad & \{\alpha_1 = \alpha_4,\ \alpha_2 = \alpha_5,\ \alpha_3 = \alpha_6, \\ & \alpha_5 = \alpha_6 \rightarrow \beta_1,\ \alpha_4 = \beta_1 \rightarrow \beta_2, \\ & \gamma_1 = \alpha_3 \rightarrow \beta_2, \\ & \gamma_2 = \alpha_2 \rightarrow \gamma_1, \\ & \gamma_3 = \alpha_4 \rightarrow \gamma_2\} \end{aligned}$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \quad \dfrac{\dfrac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \dfrac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4}}{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \Rightarrow E_5}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1} \Rightarrow E_6}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2} \Rightarrow E_7}}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3} \Rightarrow E_8}$$

Kitsendused:

$$\begin{aligned} E_8 \quad = \quad & \{\alpha_1 = \alpha_4,\ \alpha_2 = \alpha_5,\ \alpha_3 = \alpha_6, \\ & \alpha_5 = \alpha_6 \rightarrow \beta_1,\ \alpha_4 = \beta_1 \rightarrow \beta_2, \\ & \gamma_1 = \alpha_3 \rightarrow \beta_2, \\ & \gamma_2 = \alpha_5 \rightarrow \gamma_1, \\ & \gamma_3 = \alpha_4 \rightarrow \gamma_2\} \end{aligned}$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \quad \dfrac{\dfrac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \dfrac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4}}{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \Rightarrow E_5}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1} \Rightarrow E_6}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2} \Rightarrow E_7}}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3} \Rightarrow E_8}$$

Kitsendused:

$$\begin{aligned} E_8 \quad = \quad & \{\alpha_1 = \alpha_4,\ \alpha_2 = \alpha_5,\ \alpha_3 = \alpha_6, \\ & \alpha_5 = \alpha_6 \rightarrow \beta_1,\ \alpha_4 = \beta_1 \rightarrow \beta_2, \\ & \gamma_1 = \alpha_6 \rightarrow \beta_2, \\ & \gamma_2 = \alpha_5 \rightarrow \gamma_1, \\ & \gamma_3 = \alpha_4 \rightarrow \gamma_2\} \end{aligned}$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \quad \dfrac{\dfrac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \dfrac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4}}{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \Rightarrow E_5}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1} \Rightarrow E_6}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2} \Rightarrow E_7}}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3} \Rightarrow E_8}$$

Kitsendused:

$$\begin{aligned} E_8 \quad = \quad & \{\alpha_1 = \alpha_4,\ \alpha_2 = \alpha_5,\ \alpha_3 = \alpha_6, \\ & \alpha_5 = \alpha_6 \to \beta_1,\ \alpha_4 = \beta_1 \to \beta_2, \\ & \gamma_1 = \alpha_6 \to \beta_2, \\ & \gamma_2 = (\alpha_6 \to \beta_1) \to \gamma_1, \\ & \gamma_3 = \alpha_4 \to \gamma_2\} \end{aligned}$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \quad \dfrac{\dfrac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \dfrac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4}}{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \Rightarrow E_5}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1} \Rightarrow E_6}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2} \Rightarrow E_7}}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3} \Rightarrow E_8}$$

Kitsendused:

$$\begin{aligned} E_8 \quad = \quad & \{\alpha_1 = \alpha_4,\ \alpha_2 = \alpha_5,\ \alpha_3 = \alpha_6, \\ & \alpha_5 = \alpha_6 \rightarrow \beta_1,\ \alpha_4 = \beta_1 \rightarrow \beta_2, \\ & \gamma_1 = \alpha_6 \rightarrow \beta_2, \\ & \gamma_2 = (\alpha_6 \rightarrow \beta_1) \rightarrow \gamma_1, \\ & \gamma_3 = (\beta_1 \rightarrow \beta_2) \rightarrow \gamma_2\} \end{aligned}$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \quad \dfrac{\dfrac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \dfrac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4}}{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \Rightarrow E_5}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1} \Rightarrow E_6}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2} \Rightarrow E_7}}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3} \Rightarrow E_8}$$

Kitsendused:

$$\begin{aligned} E_8 \quad = \quad & \{\alpha_1 = \alpha_4,\ \alpha_2 = \alpha_5,\ \alpha_3 = \alpha_6, \\ & \alpha_5 = \alpha_6 \rightarrow \beta_1,\ \alpha_4 = \beta_1 \rightarrow \beta_2, \\ & \gamma_1 = \alpha_6 \rightarrow \beta_2, \\ & \gamma_2 = (\alpha_6 \rightarrow \beta_1) \rightarrow \alpha_6 \rightarrow \beta_2, \\ & \gamma_3 = (\beta_1 \rightarrow \beta_2) \rightarrow \gamma_2\} \end{aligned}$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{x^{\alpha_1} \in \Gamma}}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \quad \dfrac{\dfrac{}{y^{\alpha_2} \in \Gamma}}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \dfrac{}{z^{\alpha_3} \in \Gamma}}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4}}{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \Rightarrow E_5}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1} \Rightarrow E_6}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2} \Rightarrow E_7}}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3} \Rightarrow E_8}$$

Kitsendused:

$$\begin{aligned} E_8 \quad = \quad & \{\alpha_1 = \alpha_4,\ \alpha_2 = \alpha_5,\ \alpha_3 = \alpha_6, \\ & \alpha_5 = \alpha_6 \to \beta_1,\ \alpha_4 = \beta_1 \to \beta_2, \\ & \gamma_1 = \alpha_6 \to \beta_2, \\ & \gamma_2 = (\alpha_6 \to \beta_1) \to \alpha_6 \to \beta_2, \\ & \gamma_3 = (\beta_1 \to \beta_2) \to (\alpha_6 \to \beta_1) \to \alpha_6 \to \beta_2\} \end{aligned}$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \Gamma}{\Gamma \vdash x^{\alpha_4} \Rightarrow E_1} \quad \dfrac{\dfrac{y^{\alpha_2} \in \Gamma}{\Gamma \vdash y^{\alpha_5} \Rightarrow E_2} \quad \dfrac{z^{\alpha_3} \in \Gamma}{\Gamma \vdash z^{\alpha_6} \Rightarrow E_3}}{\Gamma \vdash (y^{\alpha_5} z^{\alpha_6})^{\beta_1} \Rightarrow E_4}}{x^{\alpha_1}, y^{\alpha_2}, z^{\alpha_3} \vdash (x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2} \Rightarrow E_5}}{x^{\alpha_1}, y^{\alpha_2} \vdash (\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1} \Rightarrow E_6}}{x^{\alpha_1} \vdash (\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2} \Rightarrow E_7}}{\vdash (\lambda x^{\alpha_1}.(\lambda y^{\alpha_2}.(\lambda z^{\alpha_3}.(x^{\alpha_4}(y^{\alpha_5} z^{\alpha_6})^{\beta_1})^{\beta_2})^{\gamma_1})^{\gamma_2})^{\gamma_3} \Rightarrow E_8}$$

Tuletatud tüüp:

$$\gamma_3 \quad = \quad (\beta \rightarrow \gamma) \rightarrow (\alpha \rightarrow \beta) \rightarrow \alpha \rightarrow \gamma$$

Võrrandite lahendamine unifitseerimisega

- Võrrandeid saab lahendada lihtsustusreeglite korduva rakendamise kaudu.
- Kaks peamist reeglit:
  - asenda võrdus kujul $\tau_1 \to \tau_2 = \tau_3 \to \tau_4$ kahe võrdusega $\tau_1 = \tau_3$ ja $\tau_2 = \tau_4$;
  - olgu võrdus kujul $\alpha = \tau$. Kui $\alpha \in \mathrm{FV}(\tau)$ siis raporteeri viga, vastasel korral asenda kõigis võrdustes α asemel τ.
- Abireeglid:
  - eemalda reeglid kujul $\alpha = \alpha$, $\mathrm{Bool} = \mathrm{Bool}$, jne.;
  - asenda $\tau = \alpha$ võrdusega $\alpha = \tau$;
  - kui leidub võrdus $\tau_1 = \tau_2$, kus peatüübikonstruktorid on erinevad (näit. $\mathrm{Bool} = \alpha_1 \to \alpha_2$), siis raprorteeri viga.

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$\vdash ((\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}(\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6})^{\alpha_7}$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{}{\vdash (\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}} \qquad \dfrac{}{\vdash (\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6}}}{\vdash ((\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}(\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6})^{\alpha_7}}$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{}{x^{\alpha_1} \vdash x^{\alpha_2}}}{\vdash (\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}} \qquad \dfrac{\dfrac{}{x^{\alpha_4} \vdash x^{\alpha_5}}}{\vdash (\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6}}}{\vdash ((\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}(\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6})^{\alpha_7}}$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1}}{\vdash (\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}} \qquad \dfrac{\dfrac{x^{\alpha_4} \in \{x^{\alpha_4}\}}{x^{\alpha_4} \vdash x^{\alpha_5} \Rightarrow E_2}}{\vdash (\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6}}}{\vdash ((\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}(\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6})^{\alpha_7}}$$

Kitsendused:

$$\begin{aligned} E_1 &= \{\alpha_1 = \alpha_2\} \\ E_2 &= \{\alpha_4 = \alpha_5\} \end{aligned}$$

Tüübituletus $\lambda\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1}}{\vdash (\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3} \Rightarrow E_3} \qquad \dfrac{\dfrac{x^{\alpha_4} \in \{x^{\alpha_4}\}}{x^{\alpha_4} \vdash x^{\alpha_5} \Rightarrow E_2}}{\vdash (\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6} \Rightarrow E_4}}{\vdash ((\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}(\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6})^{\alpha_7}}$$

Kitsendused:

$$\begin{array}{lcl} E_1 & = & \{\alpha_1 = \alpha_2\} \\ E_2 & = & \{\alpha_4 = \alpha_5\} \\ E_3 & = & \{\alpha_3 = \alpha_1 \rightarrow \alpha_2\} \cup E_1 \\ E_4 & = & \{\alpha_6 = \alpha_4 \rightarrow \alpha_5\} \cup E_2 \end{array}$$

Tüübituletus $\lambda\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{}{x^{\alpha_1} \in \{x^{\alpha_1}\}}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1}}{\vdash (\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3} \Rightarrow E_3} \qquad \dfrac{\dfrac{\dfrac{}{x^{\alpha_4} \in \{x^{\alpha_4}\}}}{x^{\alpha_4} \vdash x^{\alpha_5} \Rightarrow E_2}}{\vdash (\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6} \Rightarrow E_4}$$

$$\vdash ((\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}(\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6})^{\alpha_7} \Rightarrow E_5$$

Kitsendused:

$$\begin{aligned}
E_1 &= \{\alpha_1 = \alpha_2\} \\
E_2 &= \{\alpha_4 = \alpha_5\} \\
E_3 &= \{\alpha_3 = \alpha_1 \rightarrow \alpha_2\} \cup E_1 \\
E_4 &= \{\alpha_6 = \alpha_4 \rightarrow \alpha_5\} \cup E_2 \\
E_5 &= \{\alpha_3 = \alpha_6 \rightarrow \alpha_7\} \cup E_3 \cup E_4
\end{aligned}$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1}}{\vdash (\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3} \Rightarrow E_3} \qquad \dfrac{\dfrac{x^{\alpha_4} \in \{x^{\alpha_4}\}}{x^{\alpha_4} \vdash x^{\alpha_5} \Rightarrow E_2}}{\vdash (\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6} \Rightarrow E_4}}{\vdash ((\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}(\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6})^{\alpha_7} \Rightarrow E_5}$$

Kitsendused:

$$\begin{aligned} E_5 \quad = \quad & \{\alpha_1 = \alpha_2,\ \alpha_4 = \alpha_5, \\ & \alpha_3 = \alpha_1 \rightarrow \alpha_2, \\ & \alpha_6 = \alpha_4 \rightarrow \alpha_5, \\ & \alpha_3 = \alpha_6 \rightarrow \alpha_7\} \end{aligned}$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1}}{\vdash (\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3} \Rightarrow E_3} \qquad \dfrac{\dfrac{x^{\alpha_4} \in \{x^{\alpha_4}\}}{x^{\alpha_4} \vdash x^{\alpha_5} \Rightarrow E_2}}{\vdash (\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6} \Rightarrow E_4}}{\vdash ((\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}(\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6})^{\alpha_7} \Rightarrow E_5}$$

Kitsendused:

$$\begin{aligned} E_5 \quad = \quad \{ \quad & \alpha_4 = \alpha_5, \\ & \alpha_3 = \alpha_2 \rightarrow \alpha_2, \\ & \alpha_6 = \alpha_4 \rightarrow \alpha_5, \\ & \alpha_3 = \alpha_6 \rightarrow \alpha_7\} \end{aligned}$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1}}{\vdash (\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3} \Rightarrow E_3} \qquad \dfrac{\dfrac{x^{\alpha_4} \in \{x^{\alpha_4}\}}{x^{\alpha_4} \vdash x^{\alpha_5} \Rightarrow E_2}}{\vdash (\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6} \Rightarrow E_4}}{\vdash ((\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}(\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6})^{\alpha_7} \Rightarrow E_5}$$

Kitsendused:

$$\begin{aligned} E_5 \quad = \quad \{ & \\ & \alpha_3 = \alpha_2 \rightarrow \alpha_2, \\ & \alpha_6 = \alpha_5 \rightarrow \alpha_5, \\ & \alpha_3 = \alpha_6 \rightarrow \alpha_7\} \end{aligned}$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{}{x^{\alpha_1} \in \{x^{\alpha_1}\}}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1}}{\vdash (\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3} \Rightarrow E_3} \qquad \dfrac{\dfrac{\dfrac{}{x^{\alpha_4} \in \{x^{\alpha_4}\}}}{x^{\alpha_4} \vdash x^{\alpha_5} \Rightarrow E_2}}{\vdash (\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6} \Rightarrow E_4}$$

$$\vdash ((\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}(\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6})^{\alpha_7} \Rightarrow E_5$$

Kitsendused:

$$E_5 \quad = \quad \{$$

$$\alpha_6 = \alpha_5 \rightarrow \alpha_5,$$
$$\alpha_2 \rightarrow \alpha_2 = \alpha_6 \rightarrow \alpha_7\}$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1}}{\vdash (\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3} \Rightarrow E_3} \qquad \dfrac{\dfrac{x^{\alpha_4} \in \{x^{\alpha_4}\}}{x^{\alpha_4} \vdash x^{\alpha_5} \Rightarrow E_2}}{\vdash (\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6} \Rightarrow E_4}}{\vdash ((\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}(\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6})^{\alpha_7} \Rightarrow E_5}$$

Kitsendused:

$$\begin{aligned} E_5 \quad = \quad \{ & \\ & \alpha_2 = \alpha_6, \\ & \alpha_6 = \alpha_5 \rightarrow \alpha_5, \\ & \alpha_2 = \alpha_7 \} \end{aligned}$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1}}{\vdash (\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3} \Rightarrow E_3} \qquad \dfrac{\dfrac{x^{\alpha_4} \in \{x^{\alpha_4}\}}{x^{\alpha_4} \vdash x^{\alpha_5} \Rightarrow E_2}}{\vdash (\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6} \Rightarrow E_4}}{\vdash ((\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}(\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6})^{\alpha_7} \Rightarrow E_5}$$

Kitsendused:

$$\begin{aligned} E_5 \quad = \quad \{ & \\ & \alpha_2 = \alpha_5 \rightarrow \alpha_5, \\ & \\ & \alpha_2 = \alpha_7\} \end{aligned}$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{}{x^{\alpha_1} \in \{x^{\alpha_1}\}}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1}}{\vdash (\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3} \Rightarrow E_3} \qquad \dfrac{\dfrac{\dfrac{}{x^{\alpha_4} \in \{x^{\alpha_4}\}}}{x^{\alpha_4} \vdash x^{\alpha_5} \Rightarrow E_2}}{\vdash (\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6} \Rightarrow E_4}$$

$$\vdash ((\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}(\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6})^{\alpha_7} \Rightarrow E_5$$

Kitsendused:

$$E_5 \quad = \quad \{ \; \alpha_7 = \alpha_5 \rightarrow \alpha_5 \; \}$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$\frac{}{\vdash (\lambda x^{\alpha_1}.(x^{\alpha_2}\, x^{\alpha_3})^{\alpha_4})^{\alpha_5}}$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{}{x^{\alpha_1} \vdash (x^{\alpha_2} x^{\alpha_3})^{\alpha_4}}}{\vdash (\lambda x^{\alpha_1}.(x^{\alpha_2} x^{\alpha_3})^{\alpha_4})^{\alpha_5}}$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{}{x^{\alpha_1} \vdash x^{\alpha_2}} \qquad \dfrac{}{x^{\alpha_1} \vdash x^{\alpha_3}}}{\dfrac{x^{\alpha_1} \vdash \left(x^{\alpha_2} x^{\alpha_3}\right)^{\alpha_4}}{\vdash \left(\lambda x^{\alpha_1}.\left(x^{\alpha_2} x^{\alpha_3}\right)^{\alpha_4}\right)^{\alpha_5}}}$$

Tüübituletus $\lambda\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1} \qquad \dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_3} \Rightarrow E_2}}{\dfrac{x^{\alpha_1} \vdash (x^{\alpha_2} x^{\alpha_3})^{\alpha_4}}{\vdash (\lambda x^{\alpha_1}.(x^{\alpha_2} x^{\alpha_3})^{\alpha_4})^{\alpha_5}}}$$

Kitsendused:

$$\begin{aligned} E_1 &= \{\alpha_1 = \alpha_2\} \\ E_2 &= \{\alpha_1 = \alpha_3\} \end{aligned}$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1} \qquad \dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_3} \Rightarrow E_2}}{\dfrac{x^{\alpha_1} \vdash (x^{\alpha_2} x^{\alpha_3})^{\alpha_4} \Rightarrow E_3}{\vdash (\lambda x^{\alpha_1}.(x^{\alpha_2} x^{\alpha_3})^{\alpha_4})^{\alpha_5}}}$$

Kitsendused:

$$\begin{array}{lcl} E_1 & = & \{\alpha_1 = \alpha_2\} \\ E_2 & = & \{\alpha_1 = \alpha_3\} \\ E_3 & = & \{\alpha_2 = \alpha_3 \rightarrow \alpha_4\} \cup E_1 \cup E_2 \end{array}$$

Tüübituletus $\lambda\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1} \qquad \dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_3} \Rightarrow E_2}}{\dfrac{x^{\alpha_1} \vdash (x^{\alpha_2} x^{\alpha_3})^{\alpha_4} \Rightarrow E_3}{\vdash (\lambda x^{\alpha_1}.(x^{\alpha_2} x^{\alpha_3})^{\alpha_4})^{\alpha_5} \Rightarrow E_4}}$$

Kitsendused:

$$\begin{array}{lcl} E_1 & = & \{\alpha_1 = \alpha_2\} \\ E_2 & = & \{\alpha_1 = \alpha_3\} \\ E_3 & = & \{\alpha_2 = \alpha_3 \rightarrow \alpha_4\} \cup E_1 \cup E_2 \\ E_4 & = & \{\alpha_5 = \alpha_1 \rightarrow \alpha_4\} \cup E_3 \end{array}$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1} \qquad \dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_3} \Rightarrow E_2}}{\dfrac{x^{\alpha_1} \vdash (x^{\alpha_2} x^{\alpha_3})^{\alpha_4} \Rightarrow E_3}{\vdash (\lambda x^{\alpha_1}.(x^{\alpha_2} x^{\alpha_3})^{\alpha_4})^{\alpha_5} \Rightarrow E_4}}$$

Kitsendused:

$$\begin{aligned} E_4 \quad = \quad & \{\alpha_1 = \alpha_2,\ \alpha_1 = \alpha_3, \\ & \ \alpha_2 = \alpha_3 \rightarrow \alpha_4, \\ & \ \alpha_5 = \alpha_1 \rightarrow \alpha_4 \\ & \} \end{aligned}$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1} \qquad \dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_3} \Rightarrow E_2}}{\dfrac{x^{\alpha_1} \vdash (x^{\alpha_2} x^{\alpha_3})^{\alpha_4} \Rightarrow E_3}{\vdash (\lambda x^{\alpha_1}.(x^{\alpha_2} x^{\alpha_3})^{\alpha_4})^{\alpha_5} \Rightarrow E_4}}$$

Kitsendused:

$$\begin{aligned} E_4 \quad = \quad \{ \quad & \alpha_2 = \alpha_3, \\ & \alpha_2 = \alpha_3 \rightarrow \alpha_4, \\ & \alpha_5 = \alpha_2 \rightarrow \alpha_4 \\ \} \end{aligned}$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1} \qquad \dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_3} \Rightarrow E_2}}{\dfrac{x^{\alpha_1} \vdash (x^{\alpha_2} x^{\alpha_3})^{\alpha_4} \Rightarrow E_3}{\vdash (\lambda x^{\alpha_1}.(x^{\alpha_2} x^{\alpha_3})^{\alpha_4})^{\alpha_5} \Rightarrow E_4}}$$

Kitsendused:

$$\begin{aligned} E_4 \quad = \quad & \{ \\ & \alpha_3 = \alpha_3 \rightarrow \alpha_4, \\ & \alpha_5 = \alpha_3 \rightarrow \alpha_4 \\ & \} \end{aligned}$$

Tüübituletus $\lambda\!\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1} \qquad \dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_3} \Rightarrow E_2}}{\dfrac{x^{\alpha_1} \vdash (x^{\alpha_2} x^{\alpha_3})^{\alpha_4} \Rightarrow E_3}{\vdash (\lambda x^{\alpha_1}.(x^{\alpha_2} x^{\alpha_3})^{\alpha_4})^{\alpha_5} \Rightarrow E_4}}$$

Kitsendused:

$$\begin{aligned} E_4 \quad = \quad & \{ \\ & \alpha_3 = \alpha_3 \rightarrow \alpha_4, \\ & \alpha_5 = \alpha_3 \rightarrow \alpha_4 \\ & \} \end{aligned}$$

Error!

Tüübituletus

- Väga suure tüübiga term:

$$
\begin{array}{l}
\text{let } \mathit{pair} = \lambda xyz.z\ x\ y \text{ in} \\
\text{let } x_1 = \lambda y.\mathit{pair}\ y\ y \text{ in} \\
\text{let } x_2 = \lambda y.x_1(x_1\ y) \text{ in} \\
\text{let } x_3 = \lambda y.x_2(x_2\ y) \text{ in} \\
\text{let } x_4 = \lambda y.x_3(x_3\ y) \text{ in} \\
\text{let } x_5 = \lambda y.x_4(x_4\ y) \text{ in} \\
x_5(\lambda y.y)
\end{array}
$$