Konstantidega lambda-arvutus

- Konstantidega λ-termide süntaks:

$$
\begin{array}{lcll}
e & ::= & x & \text{muutuja} \\
 & | & c & \text{konstant} \\
 & | & (e_1\ e_2) & \text{aplikatsioon} \\
 & | & (\lambda x.\ e) & \text{abstraktsioon}
\end{array}
$$

- Iga konstandiga seotakse mingi arv δ-reduktsiooni reegleid.
- Näide: Naturaalarvud $(0, 1, 2, \ldots)$ ja liitmine $(+)$.

$$
\begin{array}{llll}
+\ 0\ 0 & \twoheadrightarrow_\delta & 0 & \qquad +\ 1\ 0 \twoheadrightarrow_\delta 1 \\
+\ 0\ 1 & \twoheadrightarrow_\delta & 1 & \qquad +\ 1\ 1 \twoheadrightarrow_\delta 2 \\
 & \ldots & & \qquad \ldots
\end{array}
$$

- δ-reeglite lisamine võib kokkuvoolavuse ära rikkuda!

Andmete esitamine λ-arvutuses

- Baaskombinaatorid

$$\begin{array}{lcl} \mathrm{I} & \equiv & \lambda x.\ x \\ \mathrm{K} & \equiv & \lambda x\ y.\ x \\ \mathrm{S} & \equiv & \lambda f\ g\ x.\ f\ x\ (g\ x) \end{array}$$

- "Astendamine"

$$\begin{array}{lcl} E^0 E' & \equiv & E' \\ E^n E' & \equiv & \underbrace{E(E(\ldots(E}_{n \text{ tükki}}\ E')\ldots)) \end{array}$$

- NB! $E^n(E\ E') \equiv E^{n+1}E' \equiv E\ (E^n E')$

Tõeväärtused

- Spetsifikatsioon

$$\begin{array}{lcl} \text{not true} & = & \text{false} \\ \text{not false} & = & \text{true} \end{array}$$

- Definitsioon

$$\begin{array}{lcll} \text{true} & \equiv & \lambda xy.\ x & (\equiv \ \text{K}) \\ \text{false} & \equiv & \lambda xy.\ y & \\ \text{not} & \equiv & \lambda t.\ t\ \text{false}\ \text{true} & \end{array}$$

- Näide:

$$\begin{array}{lcl} \text{not true} & \equiv & (\lambda t.\ t\ \text{false}\ \text{true})\ \text{true} \\ & \to & \text{true false true} \\ & \equiv & (\lambda x.\ \lambda y.\ x)\ \text{false true} \\ & \to & (\lambda y.\ \text{false})\ \text{true} \\ & \to & \text{false} \end{array}$$

Tingimuslause

- Spetsifikatsioon

$$\begin{aligned} \text{cond true } E_1\ E_2 &= E_1 \\ \text{cond false } E_1\ E_2 &= E_2 \end{aligned}$$

- Definitsioon

$$\text{cond} \equiv \lambda t\ x\ y.\ t\ x\ y$$

- Näide:

$$\begin{aligned} \text{cond false } E_1\ E_2 &\equiv (\lambda t\ x\ y.\ t\ x\ y)\ \text{false}\ E_1\ E_2 \\ &\twoheadrightarrow \text{false}\ E_1\ E_2 \\ &\equiv (\lambda x\ y.\ y)\ E_1\ E_2 \\ &\twoheadrightarrow E_2 \end{aligned}$$

Paarid ja ennikud

- Paarid

$$\begin{array}{lcl} \text{fst} & \equiv & \lambda p.\ p\ \text{true} \\ \text{snd} & \equiv & \lambda p.\ p\ \text{false} \\ (E_1, E_2) & \equiv & \lambda f.\ f\ E_1\ E_2 \end{array}$$

- Ennikud

$$\begin{array}{lcl} (E_1, \ldots, E_n) & \equiv & (E_1, (\ldots (E_{n-1}, E_n) \ldots)) \\ E \overset{n}{\downarrow} 1 & \equiv & \text{fst}\ E \\ E \overset{n}{\downarrow} 2 & \equiv & \text{fst}\ (\text{snd}\ E) \\ & \ldots & \\ E \overset{n}{\downarrow} i & \equiv & \text{fst}\ (\text{snd}^{i-1} E) \\ & \ldots & \\ E \overset{n}{\downarrow} n & \equiv & \text{snd}^{n-1}\ E \end{array}$$

Naturaalarvud

- Standardnumbrid

$$
\begin{array}{lcll}
\ulcorner 0 \urcorner & \equiv & \lambda x.\ x & (\equiv \ \mathrm{I}) \\
\ulcorner n{+}1 \urcorner & \equiv & (\mathrm{false}, \ulcorner n \urcorner) & \\
\mathrm{succ} & \equiv & \lambda n.\ (\mathrm{false}, n) & \\
\mathrm{pred} & \equiv & \lambda n.\ n\ \mathrm{false} & (\equiv \ \mathrm{snd}) \\
\mathrm{iszero} & \equiv & \lambda n.\ n\ \mathrm{true} & (\equiv \ \mathrm{fst})
\end{array}
$$

- Liitmine (?!)

$$\mathrm{add} \quad = \quad \lambda x\ y.\ \mathrm{cond}(\mathrm{iszero}\ x)\ y\ (\mathrm{add}(\mathrm{pred}\ x)(\mathrm{succ}\ y))$$