Naturaalarvud

- Standardnumbrid

$$
\begin{array}{lcll}
\ulcorner 0 \urcorner & \equiv & \lambda x.\ x & (\equiv \ \mathrm{I}) \\
\ulcorner n{+}1 \urcorner & \equiv & (\mathrm{false}, \ulcorner n \urcorner) & \\
\mathrm{succ} & \equiv & \lambda n.\ (\mathrm{false}, n) & \\
\mathrm{pred} & \equiv & \lambda n.\ n\ \mathrm{false} & (\equiv \ \mathrm{snd}) \\
\mathrm{iszero} & \equiv & \lambda n.\ n\ \mathrm{true} & (\equiv \ \mathrm{fst})
\end{array}
$$

- Liitmine (?!)

$$\mathrm{add} \quad = \quad \lambda x\ y.\ \mathrm{cond}(\mathrm{iszero}\ x)\ y\ (\mathrm{add}(\mathrm{pred}\ x)(\mathrm{succ}\ y))$$

Naturaalarvud

- Church'i numbrid

$$\begin{array}{lcl} \underline{n} & \equiv & \lambda f\ x.\ f^n\ x \\ \text{succ} & \equiv & \lambda n.\ \lambda f\ x.\ n\ f\ (f\ x) \\ \text{iszero} & \equiv & \lambda n.\ n\ (\lambda x.\ \text{false})\ \text{true} \\ \text{add} & \equiv & \lambda m\ n.\ \lambda f\ x.\ m\ f\ (n\ f\ x) \end{array}$$

- Näide

$$\begin{array}{lcl} \text{add}\ \underline{2}\ \underline{1} & \equiv & (\lambda m\ n.\ \lambda f\ x.\ m\ f\ (n\ f\ x))\ \underline{2}\ \underline{1} \\ & \twoheadrightarrow & \lambda f\ x.\ \underline{2}\ f\ (\underline{1}\ f\ x) \\ & \twoheadrightarrow & \lambda f\ x.\ f\ (f\ (\underline{1}\ f\ x)) \\ & \twoheadrightarrow & \lambda f\ x.\ f\ (f\ (f\ x))) \\ & \equiv & \underline{3} \end{array}$$

- Korrutamine ja astendamine

$$\begin{array}{lcl} \text{mul} & \equiv & \lambda m\ n.\ \lambda f\ x.\ m\ (n\ f)\ x \\ \text{exp} & \equiv & \lambda m\ n.\ \lambda f\ x.\ n\ m\ f\ x \end{array}$$

Naturaalarvud

- Ühe lahutamine — abifunktsiooni spetsifikatsioon

$$\begin{array}{lcl}
\text{prefn } f\ (\text{true}, x) & = & (\text{false}, x) \\
\text{prefn } f\ (\text{false}, x) & = & (\text{false}, f\ x) \\
(\text{prefn } f)^n\ (\text{false}, x) & = & (\text{false}, f^n\ x) \\
(\text{prefn } f)^n\ (\text{true}, x) & = & (\text{false}, f^{n-1}\ x)
\end{array}$$

- Ühe lahutamine — definitsioon

$$\begin{array}{lcl}
\text{prefn} & \equiv & \lambda f\ p.\ (\text{false}, (\text{cond}\ (\text{fst}\ p)\ (\text{snd}\ p)\ (f\ (\text{snd}\ p)))) \\
\text{pred} & \equiv & \lambda n.\ \lambda f\ x.\ \text{snd}\ (n\ (\text{prefn}\ f)\ (\text{true}, x))
\end{array}$$

- Näide

$$\begin{array}{lcl}
\text{pred}\ \underline{n}\ f\ x & \twoheadrightarrow & \text{snd}\ (\underline{n}\ (\text{prefn}\ f)\ (\text{true}, x)) \\
& \twoheadrightarrow & \text{snd}\ ((\text{prefn}\ f)^n\ (\text{true}, x) \\
& \twoheadrightarrow & \text{snd}\ (\text{false}, f^{n-1}\ x) \\
& \twoheadrightarrow & f^{n-1}\ x
\end{array}$$