

# Seminar in Tartu, Estonia

## Designated Verifier Signatures: Attacks, New Definitions and Constructions



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# Outline

- Motivation for DVS
- Attacks on Some Previous Constructions
- New Security Notions
- Our Own Construction
- Conclusion

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# Motivation



I w4nt 2 read s0me b00k.  
But I h4ve 2 b a subscr1b3r!  
Th1s 1s ok, I c4n s1gn my request  
But 1 do not w4nt S11ck to show  
the s1gnatur3 2 oth3rs!



# Motivation



I w4nt 2 read s0me b00k.  
But I h4ve 2 b a subscr1b3r!  
Th1s 1s ok, I c4n s1gn my request  
But 1 do not w4nt S11ck to show  
the s1gnatur3 2 oth3rs!

My fr1end Markus sa1d I can  
us3 des1nated ver1f1er s1gnatures!  
S1nce Desmond can s1mulate such  
s1gnatures, the s1gnatures are  
non-transferable.



Hej! I am Markus.

## More applications?

- E-voting: Signy is a voter, Desmond is a tallier. Desmond gets to know voter is Signy but cannot prove it to anybody else.
- Also related to privacy-preserving data-mining:
  - ★ Desmond knows Signy is a loyal customer; Signy gets bonus
  - ★ Desmond can add information about Signy in the database and process it later
  - ★ Desmond can't prove to anybody else that the database is correct but he trusts himself!
- Etc etc etc

# Intermission: proofs of knowledge

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- Three-message protocols: on secret inputs  $x_S$  and  $x_D$  and a common input  $(g, \dots, y_S, y_D)$ , Signy sends  $m_1$ , Desmond sends  $m_2$ , Signy sends  $m_3$ . Desmond either rejects or accepts.
- Completeness: if Signy “knows”  $x_S$ , Desmond accepts.
- Proof of knowledge: if Signy does not “know”  $x_S$ , Desmond rejects (w.h.p.).
- Fiat-Shamir heuristic: Signy sends  $(m_1, H(\dots, m_1), m_3)$  to Desmond. Desmond accepts or rejects. Correct if  $H$  is a *random oracle*.

# Thus spake Markus to Signy:

Public key  $y_S = g^{x_S}$



Public key  $y_D = g^{x_D}$





# Thus spake Markus to Signy:

Public key  $y_S = g^{x_S}$   
Generate  $s \leftarrow m^{x_S}$

Signy does



Public key  $y_D = g^{x_D}$



# Thus spake Markus to Signy:

Public key  $y_S = g^{x_S}$

Signy does

Generate  $s \leftarrow m^{x_S}$

Generate random  $w, t, r \leftarrow \mathbb{Z}_q$



Public key  $y_D = g^{x_D}$



# Thus spake Markus to Signy:

Public key  $y_S = g^{x_S}$



Signy does  
Generate  $s \leftarrow m^{x_S}$   
Generate random  $w, t, r \leftarrow \mathbb{Z}_q$   
Set  $h \leftarrow H(g^w y_D^t, g^r, m^r)$

Public key  $y_D = g^{x_D}$




# Thus spake Markus to Signy:

Public key  $y_S = g^{x_S}$

Signy does

Generate  $s \leftarrow m^{x_S}$   
Generate random  $w, t, r \leftarrow \mathbb{Z}_q$   
Set  $h \leftarrow H(g^w y_D^t, g^r, m^r)$   
Set  $z \leftarrow r + (h + w)x_S$

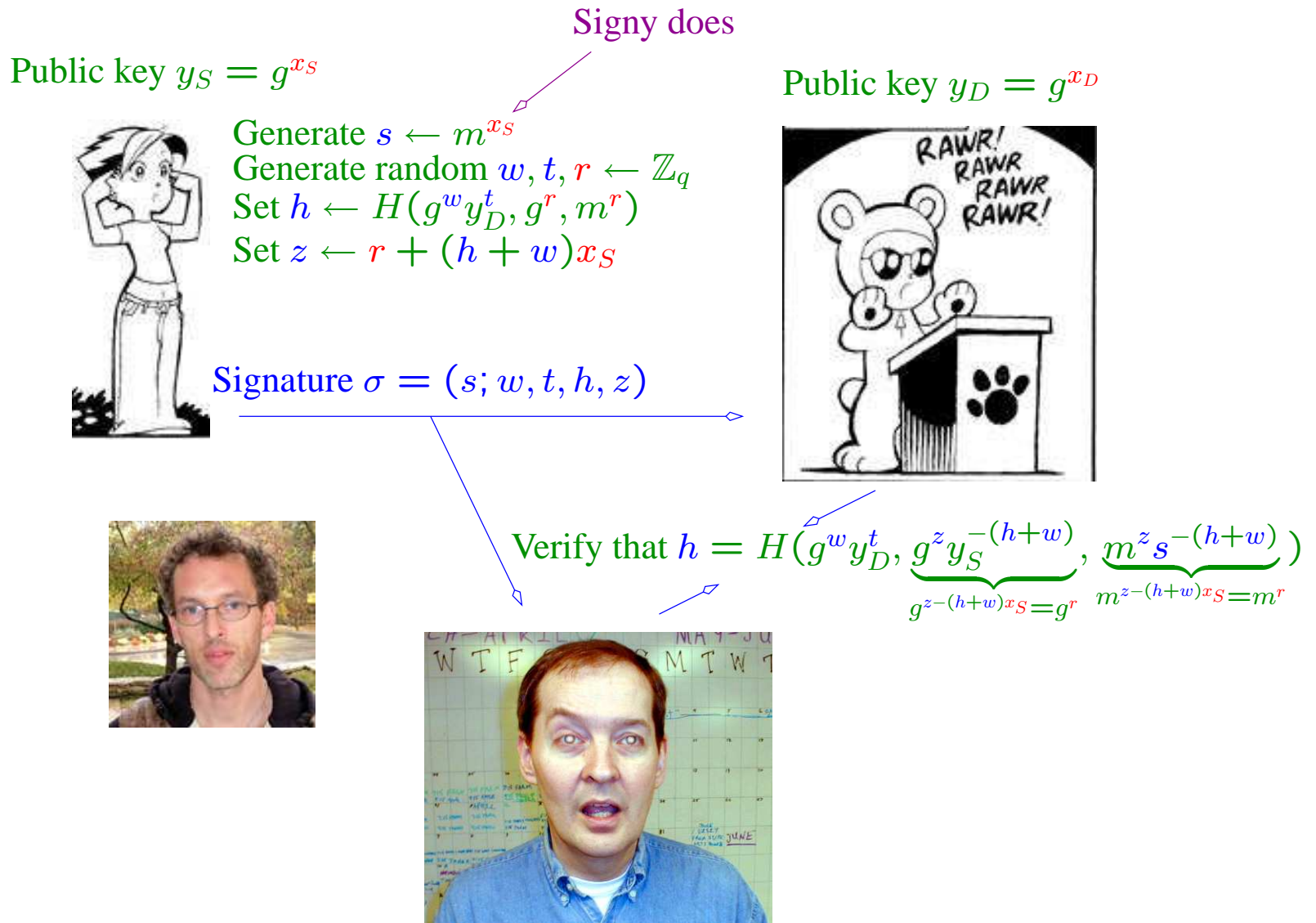
Signature  $\sigma = (s; w, t, h, z)$



Public key  $y_D = g^{x_D}$



# Thus spake Markus to Signy:



# Thus spake Markus to Desmond:

Desmond does

Public key  $y_S = g^{x_S}$



Choose any  $s$   
 Generate random  $z, \alpha, \beta \leftarrow \mathbb{Z}_q$   
 Set  $h \leftarrow H(g^\alpha, g^z y_S^{-\beta}, m^z s^{-\beta})$   
 Set  $w \leftarrow \beta - h, t \leftarrow (\alpha - w)x_D^{-1}$

Signature  $\sigma = (s; w, t, h, z)$

Public key  $y_D = g^{x_D}$



Verify that  $h = H(\underbrace{g^w y_D^t}_{g^{w+t x_D}}, \underbrace{g^z y_S^{-(h+w)}}_{g^z y_S^{-\beta}}, \underbrace{m^z s^{-(h+w)}}_{m^z s^{-\beta}})$



Das ist ja Korrekt!

## Thus spake Markus to both:

- If Signy signs:  $s = m^{x_S}$ , thus  $(g, y_S, m, s)$  is a DDH tuple.
  - ★  $(g, y_S, m, s) = (g, g^a, g^b, g^{ab})$  for some  $a, b$
- Signy proves in NIZK that  $(g, y_S, m, s)$  is a DDH tuple.
- If Desmond simulates:  $\bar{s}$  is chosen randomly, thus  $(g, y_S, m, \bar{s})$  is not a DDH tuple with very high probability,  $1 - \frac{1}{q}$ 
  - ★  $c = g^w y_D^t$  for which Desmond knows the trapdoor  $x_D$
  - ★ Desmond “simulates” proof by using the trapdoor *for any*  $\bar{s} \in \mathbb{Z}_p$
- Signy can disavow, w.h.p.  $1 - \frac{1}{q}$ , by proving that  $\bar{s} \neq m^{x_S}$

## Thus spake Markus to both:

- To generate a valid  $\sigma \leftarrow (s; w, t, h, z)$  you must know either  $x_S$  or  $x_D$
- Thus Desmond knows  $\sigma$  was generated by Signy
  - ★ Since Desmond did not generate it himself
- Any third party doesn't know whether  $\sigma$  was generated by Signy or Desmond

And Signy was very happy and Desmond covered in snow.



## But Desmond met Guilin and Guilin spake to him:



Heh-heh!  
No plobrem!  
I wirr bleak that!

# But Desmond met Guilin and Guilin spake to him:

Public key  $y_S = g^{x_S}$



Generate random  $w, t, r \neq \bar{r} \leftarrow \mathbb{Z}_q$   
 Set  $h \leftarrow H(g^w y_D, g^r, m^{\bar{r}})$   
 Set  $z \leftarrow r + (h + w)x_S$   
 Set  $\bar{s} \leftarrow m^{x_S} \cdot m^{(r-\bar{r})/(h+w)}$

Signature  $\sigma = (\bar{s}; w, t, h, z)$

Signy can also do this!



Verify that  $h = H(g^w y_D, \underbrace{g^z y_S^{-(h+w)}}_{g^{z-(r-\bar{r})} = g^r}, \underbrace{m^z (\bar{s})^{-(h+w)}}_{m^{z-(h+w)x_S-(r-\bar{r})} = m^{\bar{r}}})$

Public key  $y_D = g^{x_D}$



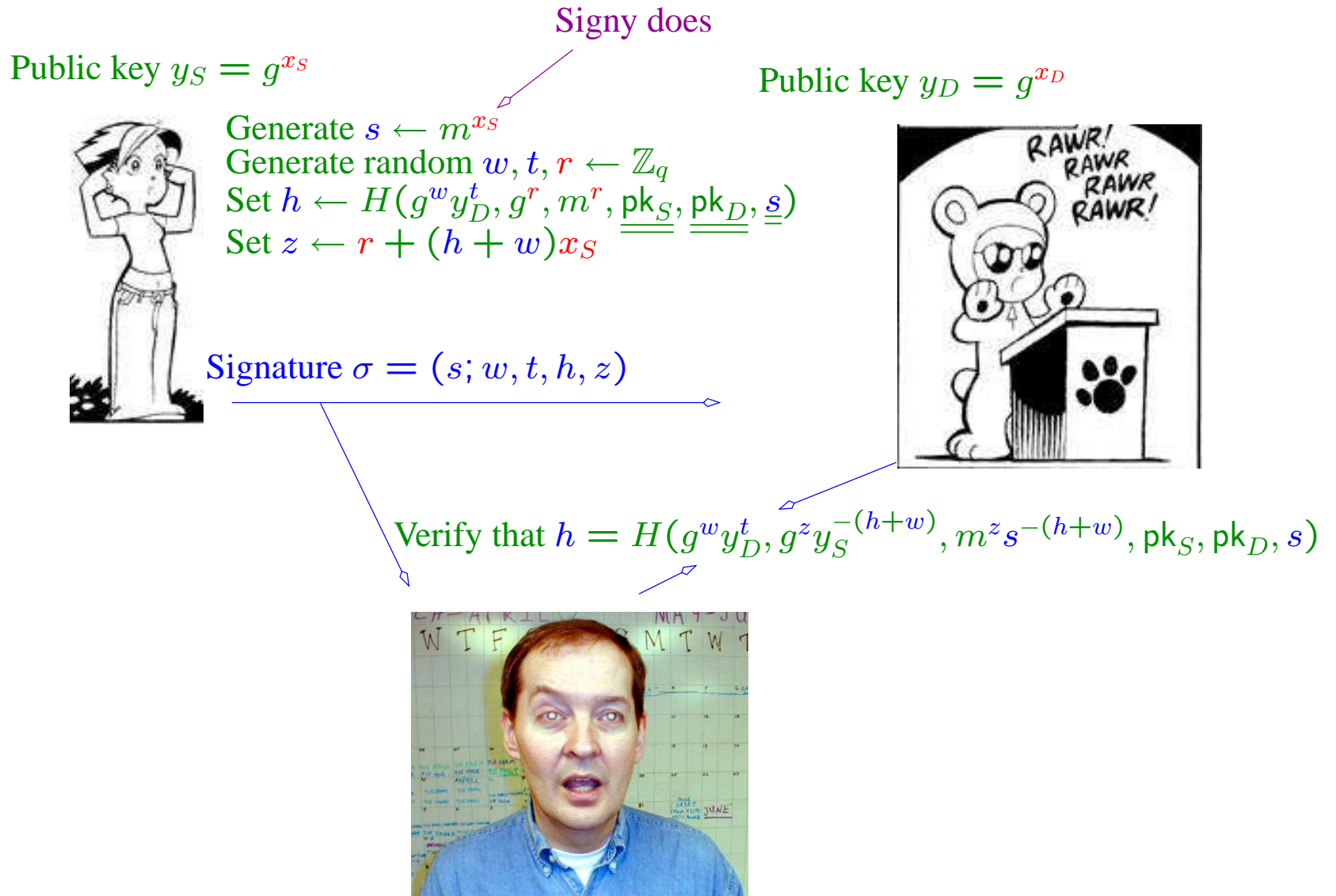
## But Desmond met Guilin and Guilin spake to him:

- Verification succeeds, thus Desmond accepts it as Signy's signature
- However, since  $\bar{s} \neq m^{x_S}$ , Signy can later disavow it!

And Desmond was not so happy anymore.



# Quick fix:



# Then, Signy met some other people

- Steinfeld, Bull, Wang and Pieprzyk said: use a bilinear pairing  $\langle \cdot, \cdot \rangle$ 
  - ★  $\langle b^a, d^c \rangle = \langle b, d \rangle^{ac}$  with natural hardness assumptions
- Signy signs  $m$ :  $s = \langle m^{x_S}, y_D \rangle = \langle m, g \rangle^{x_S x_D}$
- Desmond simulates:  $\bar{s} = \langle m^{x_D}, y_S \rangle = \langle m, g \rangle^{x_S x_D}$
- Verification by Desmond:  $\langle m, y_S^{x_D} \rangle = s?$

And Signy was happy again and kissed Pieprzyk.



I like this job!

## However, Desmond met Guilin again

Guilin spake to Desmond:

- Signy can compute  $y_{SD} := g^{x_S x_D}$  and publish it
- Then anybody can sign  $m$  as  $s = \langle m, y_{SD} \rangle = \langle m, g \rangle^{x_S x_D}$
- Thus Signy can delegate her subscription to your library, without revealing her public key

And Desmond wanted to cry.

## And so forth and so forth

- Signy and Desmond met many wise men who proposed better and better designated verifier signature schemes.
- However, Guilin broke them all!
- Sad story, eh?
- Signy even thought about never reading a book again!

# What went wrong?

- [JSI1996]: disavowability claimed but does not exist
  - [SBWP2003] and some other schemes were delegatable
- ⇒ propose a modification that is *unforgeable*
- ★ Use as tight reductions as possible
  - ★ ...and as weak trust model as possible
- ⇒ Eliminate disavowal or make it “secure”
- *Non-delegatability* was never considered before
- ⇒ Define non-delegatability and propose a non-delegatable scheme



# Unforgeability

Consider the next game:

- Choose random key pairs for Signy and Desmond
- Give the Forger both public keys, an oracle access to Signy's signing algorithm, Desmond's simulation algorithm and the hash function
- Forger returns a message  $m$  and a signature  $\sigma$

Forger is *successful* if verification on  $(m, \sigma)$  succeeds and he never asked a sign/simul query on  $m$  that returned  $\sigma$

Scheme is  $(\tau, q_h, q_s, \varepsilon)$ -*unforgeable*  $\iff$  no  $(\tau, q_h, q_s)$ -forger has success probability  $> \varepsilon$

Forger runs in time  $\tau$ , does  $q_h$  queries to hash function and  $q_s$  queries to either signing or simulation algorithm

# Non-Transferability

- A scheme is *perfectly* non-transferable if signatures generated by Signy and Desmond come from the same distribution.
  - ★ Perfectly non-transferable schemes *cannot* have disavowal protocols!
  - ★ As we showed, JSI is perfectly non-transferable!
- A scheme is *computationally* non-transferable if signatures generated by Signy and Desmond come from distributions that are computationally indistinguishable.
  - ★ Computationally non-transferable schemes *may* have a trapdoor that can be used for constructing disavowal protocols

# Non-Delegatability

Requirement: if Forger produces valid signatures with probability  $> \kappa$  then he knows either the secret key of Signy or the secret key of Desmond

We require there exists a knowledge extractor such that

- **If** a Forger produces a valid signature  $\sigma$  on  $m$  w.p.  $\varepsilon > \kappa$   
**then** knowledge extractor, given  $(m$  and oracle access to Forger on input  $m$ , produces one of the two secret keys in time  $\frac{\tau}{\varepsilon - \kappa}$ .

Then the scheme is  $(\tau, \kappa)$ -non-delegatable.

Note: this is a standard “proof of knowledge” requirement.

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# Underlying Idea of Our Scheme

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- If Signy signs: she proves that her public key  $(g_1, g_2, y_{1S} = g_1^{x_S}, y_{2S} = g_2^{x_S})$  is a DDH tuple.
- We again employ  $c = g^w y_D^t$  (trapdoor commitment) for which Desmond knows the trapdoor  $x_D$ , thus the proof is designated-verifier.
- Desmond simulates this proof by using the trapdoor information
- Signy cannot disavow since there is perfect non-transferability

# And Thus We Spake to Signy:

Signy does

Public key  $(y_{1S} = g_1^{x_S}, y_{2S} = g_2^{x_S})$



Generate random  $w, t, r \leftarrow \mathbb{Z}_q$   
 Set  $h \leftarrow H(pk_S, pk_D, g_1^w y_{1D}^t, g_1^r, g_2^r, m)$   
 Set  $z \leftarrow r + (h + w)x_S$

Signature  $\sigma = (w, t, h, z)$

Public key  $(y_{1D} = g_1^{x_D}, y_{2D} = g_2^{x_D})$



Verify that  $h = H(pk_S, pk_D, g_1^w y_{1D}^t, g_1^z y_{1S}^{-(h+w)}, g_2^z y_{2S}^{-(h+w)}, m)$



# And Thus We Spake to Desmond:

Desmond does

Public key  $(y_{1S} = g_1^{x_S}, y_{2S} = g_2^{x_S})$



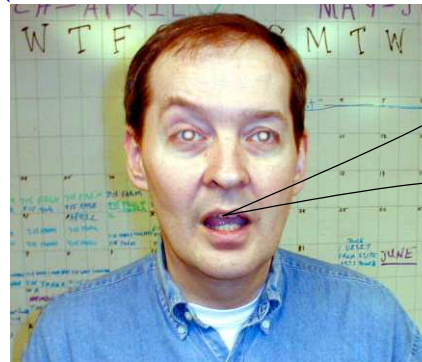
Choose any  $s$   
 Generate random  $z, \alpha, \beta \leftarrow \mathbb{Z}_q$   
 Set  $h \leftarrow H(\text{pk}_S, \text{pk}_D, g_1^\alpha, g_1^z y_{1S}^{-\beta}, g_2^z y_{2S}^{-\beta}, m)$   
 Set  $w \leftarrow \beta - h, t \leftarrow (\alpha - w)x_D^{-1}$

Signature  $\sigma = (w, t, h, z)$

Public key  $(y_{1D} = g_1^{x_D}, y_{2D} = g_2^{x_D})$



Verify that  $h = H(\text{pk}_S, \text{pk}_D, g_1^w y_{1D}^t, g_1^z y_{1S}^{-(h+w)}, g_2^z y_{2S}^{-(h+w)}, m)$



Das ist ja Korrekt!

# Properties of The New Scheme

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- Twice longer public keys than in JSI — enables to get tight unforgeability reductions
  - ★ In *non-programmable random oracle model*
- No disavowal
  - ★ Orthogonal to the security requirements of an DVS scheme
- Non-delegatability: proven, but the reduction is not tight



# Unforgeability

**Theorem.** Let  $G$ ,  $|G| = q$  be a  $(\tau', \varepsilon')$ -DDH group. The proposed scheme is  $(\tau, q_h, q_s, \varepsilon)$ -unforgeable in the non-programmable random oracle model with  $\tau \leq \tau' - (3.2q_s + 5.6)t_{\text{exp}}$  and  $\varepsilon \geq \varepsilon' + q_s q_h q^{-2} + q^{-1} + q_h q^{-2}$ .

**Proof sketch:** Adversary  $A$  has to solve DDH on input  $(g_1, g_2, y_{1D}, y_{2D})$ . Set this to Desmond's public key, and set Signy's public key to be equal to a random DDH tuple (for which  $A$  knows the corresponding secret key). Give  $A$  an oracle access to Forger. Answer all hash queries truthfully (but store them). Answer all signing and simulation queries by following Signy's algorithm. (Possible since  $A$  knows Signy's secret key.) It comes out that  $A$  works in time and with success probability, claimed above.

**Note:** This is a tight reduction. In practical setting it means that whenever you can forge a signature—e.g.,  $2^{-80}$ —, you can almost always solve DDH and in comparable time.

# Non-programmable random oracle model

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RO model	NPRO model
<p data-bbox="195 289 1020 378">Environment doesn't have access to the RO.</p> <p data-bbox="195 440 1020 683"><b>In the Katz-Wang signature scheme:</b> adversary does not know signer's secret key, and thus cannot create valid signatures without defining a <math>H</math> that just satisfies the verification equation</p> <p data-bbox="195 740 1020 878"><b>Best case proof:</b> shows that for every adversary, there exists a function <math>H</math> such that the result holds</p> <p data-bbox="195 943 1020 1027">But <math>H</math> depends on Forger's actions and thus cannot be instantiated in some sense!</p>	<p data-bbox="1077 289 1759 329">Environment has access to the RO.</p> <p data-bbox="1077 440 1902 634"><b>In our scheme:</b> adversary has access to Signy's secret key, and can thus create valid signatures without redefining <math>H</math></p> <p data-bbox="1077 740 1902 829"><b>Average case proof:</b> shows that the result holds for a randomly chosen function <math>H</math></p> <p data-bbox="1077 943 1633 984"><math>H</math> can be chosen in advance</p>

# New Conventional Signature Scheme

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- Take the new DVS scheme with assumption that Desmond is a trusted third party, and his public key is a common reference string (CRS).
- That is, Signy signs  $m$  by
  - ★ Choosing random  $w, t, r \leftarrow_R \mathbb{Z}_q$
  - ★ Setting  $h \leftarrow H(\text{pk}_S, g_1^w y_{1D}^t, g_1^r, g_2^r, m)$
  - ★ Setting  $z \leftarrow r + (h + w)x_S$  and outputting  $\sigma = (w, t, h, z)$
- New signature scheme with tight security reduction to DDH problem in NPRO+CRS model

# Delegatability

**Theorem.** Let  $\kappa \geq 1/q$ . Assume that for some message  $m$ , Forger can produce signature in time  $\tau'$  and with probability  $\varepsilon \geq \kappa$ . Then there exists a knowledge extractor that on input a valid signature  $\sigma$  and on black-box oracle access to Forger (with an internal state compatible with  $\sigma$ ) can produce one of the two secret keys in expected time  $\tau \leq 56\tau'/\kappa$ .

**Note:** This is an imprecise reduction. For example, if Forger has advantage  $2^{-30}$  then Knowledge Extractor works in time  $2^{36} \cdot \tau'$ , with probability 1.

# Conclusions

- And Desmond was happy since only valid subscribers were able to borrow the books.
  - ★ And these subscribers could not delegate their subscriptions!
- And Signy was happy since Desmond could not prove that she borrowed these books.

# Any questions?



Caveat: This presentation is based on a draft version of the paper!