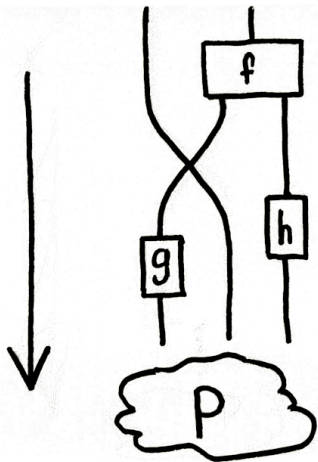


The Structure of Concurrent Process Histories

(COORDINATION 2021)

Chad Nester

Tallinn University of Technology

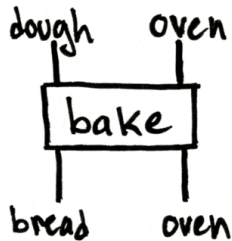
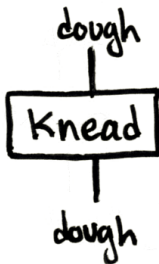
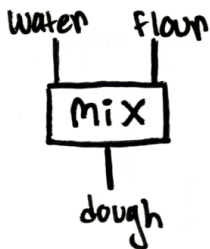


Resource Theories \Leftrightarrow Symmetric Monoidal Categories

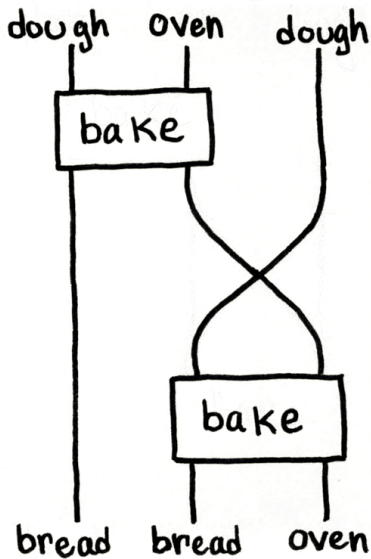
atomic objects:

{bread, dough, water, flour, oven}

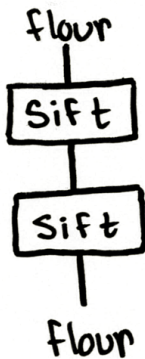
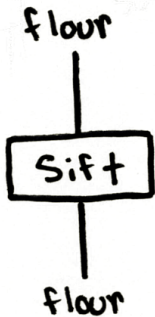
generating morphisms:



Resource Transformations \leftrightarrow String Diagrams

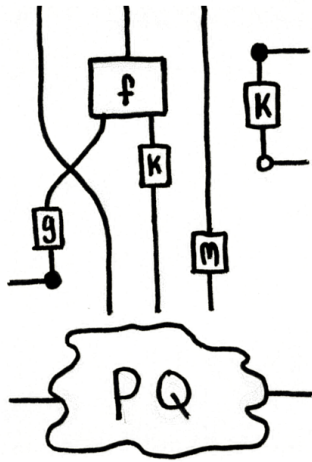
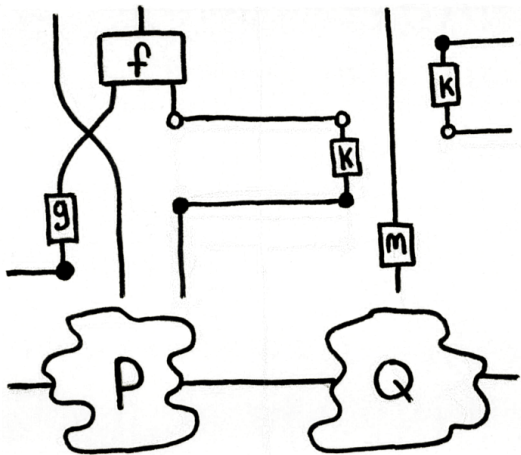


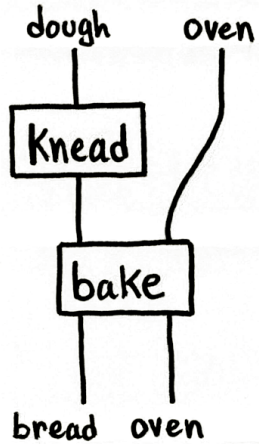
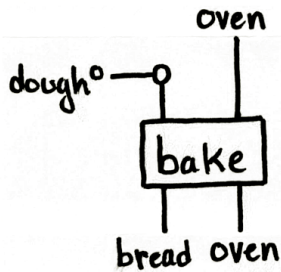
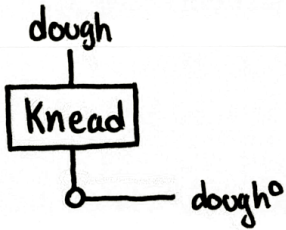
Same Effect \leftrightarrow Equal as Morphisms



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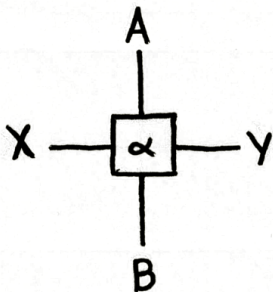




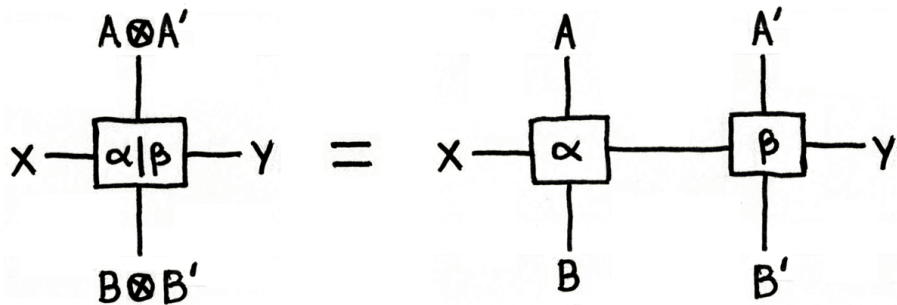


Single Object Double Categories (\mathbb{D}):

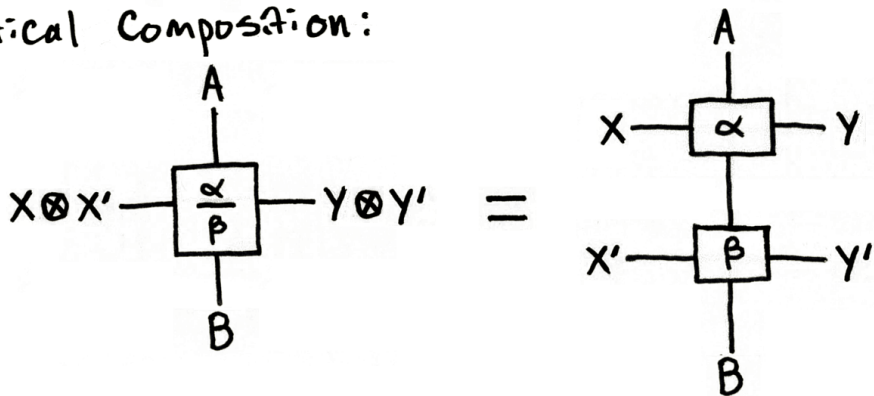
- Horizontal edge monoid: $A, B, C \in (\mathbb{D}_H, \otimes, I)$
- Vertical edge monoid: $X, Y, Z \in (\mathbb{D}_V, \otimes, I)$
- Cells:



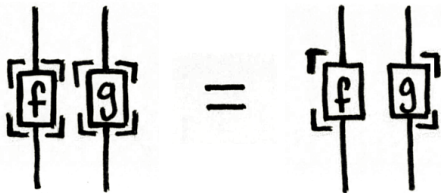
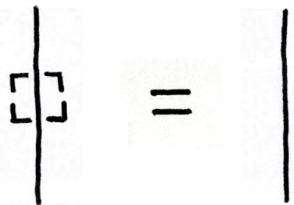
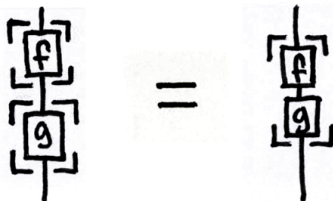
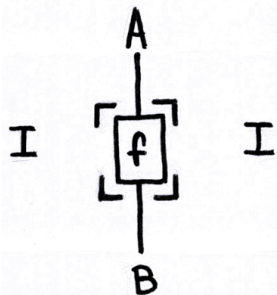
Horizontal Composition:



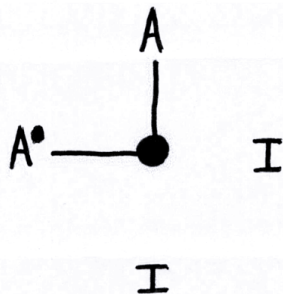
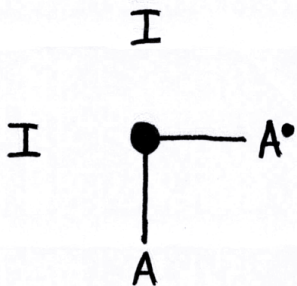
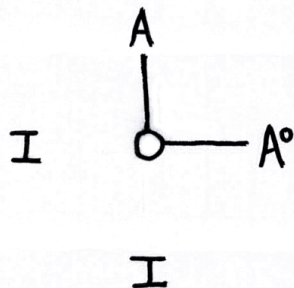
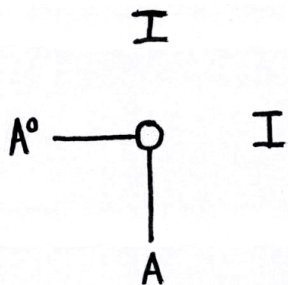
Vertical Composition:



Monoidal Category $\xrightarrow{\text{Corner Structure}}$ Double Category



Corner cells for each object A :



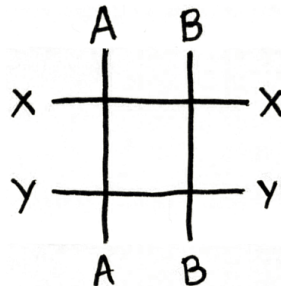
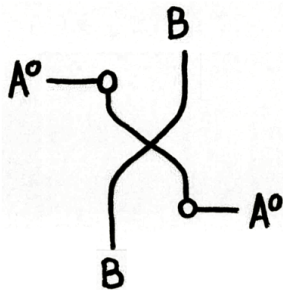
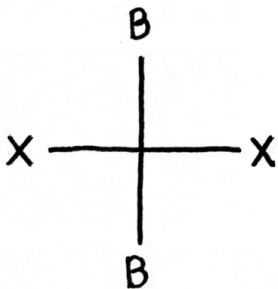
Corners must satisfy the Yanking equations:

$$\begin{array}{c} \text{---} \circ \\ | \\ \circ \text{---} \end{array} = \text{---}$$

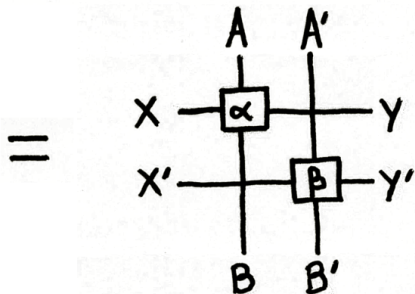
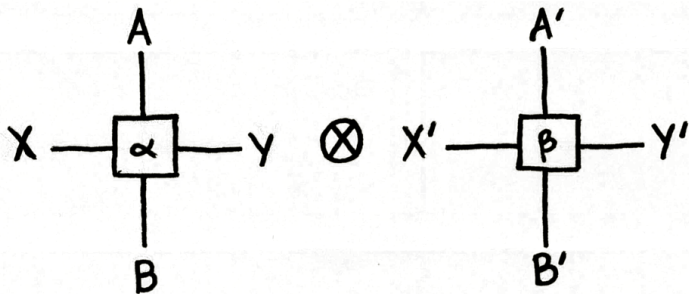
$$\text{---} = \begin{array}{c} \bullet \text{---} \\ | \\ \text{---} \bullet \end{array}$$

$$\begin{array}{c} | \\ \circ \text{---} \circ \\ | \end{array} = | = \begin{array}{c} | \\ \bullet \text{---} \bullet \\ | \end{array}$$

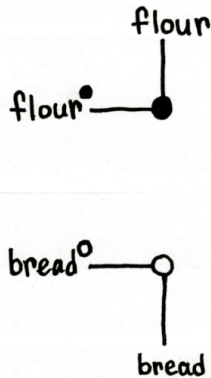
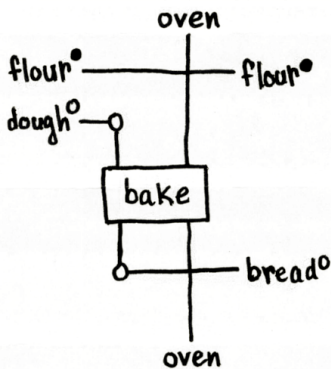
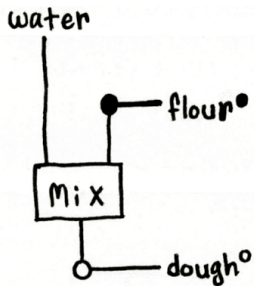
Crossing Cells arise from Corner Structure :



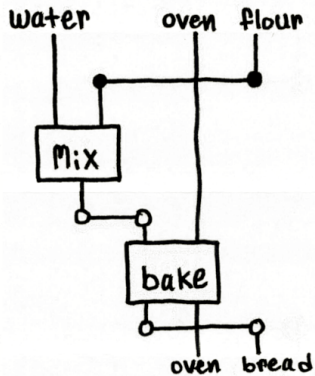
Crossing Cells \longrightarrow Monoidal Double Category



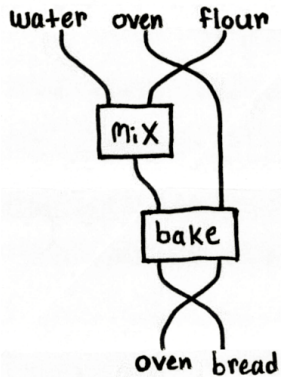
Now Cells \leftrightarrow Concurrent Resource Transformations



Horizontal Composition \leftrightarrow Interaction



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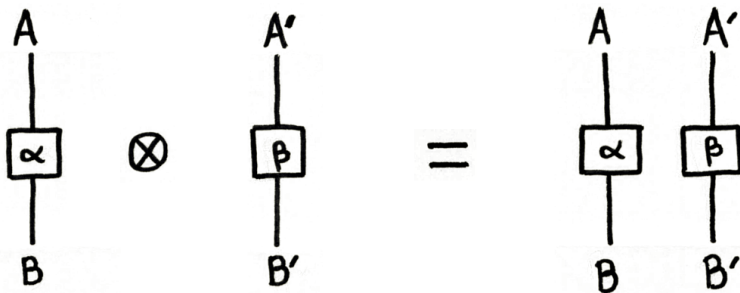
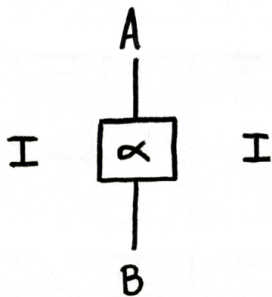


Corner structure goes by many names:

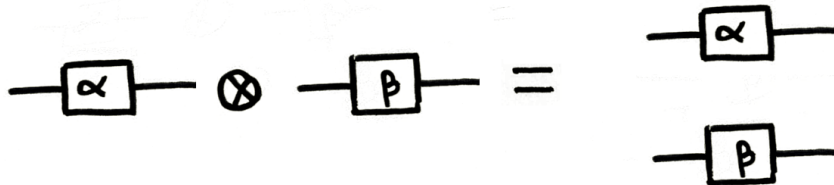
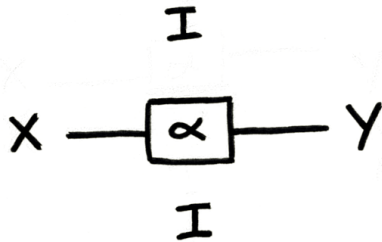
- Proarrow equipment
- Connection structure
- Companions and conjoiners
- Frames bicategory

Fundamental to formal category theory.

Vertical Cells \leftrightarrow Original Resource Theory

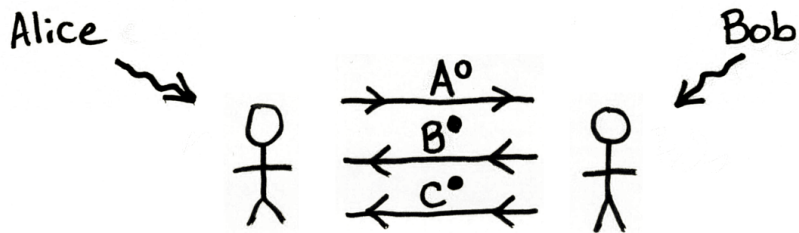


Horizontal cells \longleftrightarrow Category of Exchanges



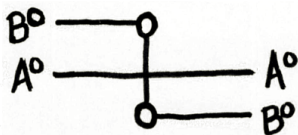
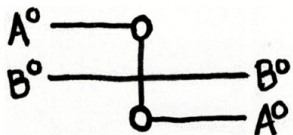
Vertical Edge Monoid \leftrightarrow Exchanges

We understand $A^\circ \otimes B^\circ \otimes C^\circ$ as in:



Isomorphic Exchanges \longleftrightarrow Same Effect

• $A^0 \otimes B^0 \cong B^0 \otimes A^0$ via



Similarly:

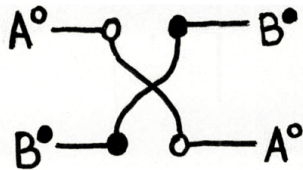
• $A^1 \otimes B^1 \cong B^1 \otimes A^1$

• $I^0 \cong I \cong I^1$

• $(A \otimes B)^0 \cong A^0 \otimes B^0$

• $(A \otimes B)^1 \cong A^1 \otimes B^1$

There is an arrow $A^\circ \otimes B^\circ \rightarrow B^\circ \otimes A^\circ$:



... but not necessarily $B^\circ \otimes A^\circ \rightarrow A^\circ \otimes B^\circ$.

\Rightarrow Horizontal Cells need not be symmetric monoidal.

Causal structure. Can't give if don't have.

Possible Future Work:

- Behavioural Types.
- Connect to Existing Approaches to modelling Concurrent Systems.
- Situated Transition Systems.
- Mystery : Why proarrow equipments?

Thanks
for
Listening!

(Ask Questions)