Graphs, 1st test (2nd try)

October 10th, 2008

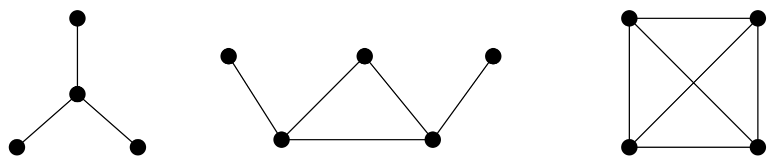
Exercise 1. How many different (up to isomorphism) 3-regular simple graphs with six vertices are there?

Exercise 2. For any $n \in \mathbb{N}$ define the simple graph $G_n = (V_n, E_n)$ as follows:

- The elements of V_n are all subsets of the set $\{1, \ldots, n\}$, except the empty set.
- Two elements $A, B \in V_n$ are neighbours if and only if $A \cap B \neq \emptyset$.

For which values of n is G_n Eulerian?

Exercise 3. Let G be a connected simple graph. Show that if the following three graphs are not induced subgraphs of G, then G has a Hamiltonian path.



Exercise 4. Consider the graph G_4 from the second exercise. For an edge e connecting the vertices A and B define its weight $w(e)$ as the sum of elements of the set $A \cap B$. Find

- a maximum-weight spanning tree;
- a minimum-weight spanning tree

of the resulting graph with edge weights.

The usage of written/printed materials is allowed.

Graafid, 1. kontrolltöö (teine katse)

8. jaanuar 2009

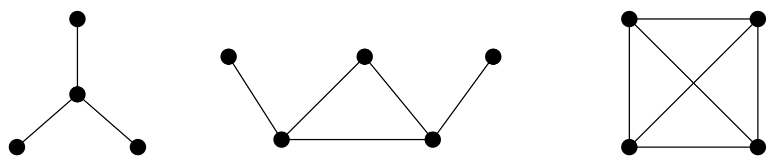
Ülesanne 1. Kui palju on olemas erinevaid (isomorfismi täpsusega) 3-regulaarseid kuuetipulisi lihtgraafe?

Ülesanne 2. Iga $n \in \mathbb{N}$ jaoks defineerime lihtgraafi $G_n = (V_n, E_n)$ järgmiselt:

- V_n elementideks on hulga $\{1, \ldots, n\}$ kõik alamhulgad, välja arvatud tühi hulk;
- Kaks tippu $A, B \in V_n$ on servaga ühendatud parajasti siis, kui $A \cap B \neq \emptyset$.

Milliste n väärtuste korral on G_n Euleri graaf?

Ülesanne 3. Olgu G sidus lihtgraaf. Näita, et kui allolevad kolm graafi ei ole G indutseeritud alamgraafideks, siis leidub G-s Hamiltoni ahel.



Ülesanne 4. Vaatame graafi G_4 teisest ülesandest. Tippe A ja B ühendava serva e jaoks defineerime tema kaalu $w(e)$ kui hulga $A \cap B$ elementide summa. Leia

- mõni maksimaalse kaaluga aluspuu;
- mõni minimaalse kaaluga aluspuu

saadud kaalutud servadega graafis.

Paberkandjal materjale tohib kasutada.