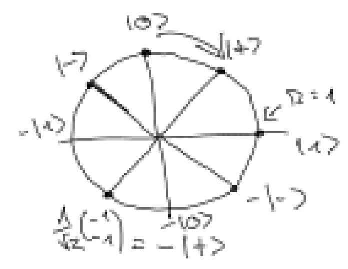
Practice Session Feb 16

$|0\rangle = \begin{pmatrix}1\\0\end{pmatrix}$ $\begin{pmatrix}0\\1\end{pmatrix} = |1\rangle$

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{pmatrix}\alpha\\\beta\end{pmatrix}$

$|\alpha|^2 + |\beta|^2 = 1$

Reals only:



$|+\rangle = \frac{1}{\sqrt2}|0\rangle + \frac{1}{\sqrt2}|1\rangle = \frac{1}{\sqrt2}\begin{pmatrix}1\\1\end{pmatrix}$

$\left|\frac{1}{\sqrt2}\right|^2 + \left|\frac{1}{\sqrt2}\right|^2 = 1$

$|-\rangle = \frac{1}{\sqrt2}(|0\rangle + |1\rangle) = \frac{1}{\sqrt2}\begin{pmatrix}1\\-1\end{pmatrix}$

$-|-\rangle$

$|+\rangle$ and $-1|+\rangle$ are physically ind.

$|+\rangle = \gamma|+\rangle$

$|\gamma| = 1$

$|00\rangle$
$|01\rangle$
$|10\rangle$
$|11\rangle$

$|\psi\rangle = \alpha_1|00\rangle + \alpha_2|01\rangle + \alpha_3|10\rangle + \alpha_4|11\rangle = \begin{pmatrix}\alpha_1\\\alpha_2\\\alpha_3\\\alpha_4\end{pmatrix}$

$\sum_i |\alpha_i|^2 = 1$

Gates

$|0\rangle$ —[ ]—

Matrices that are unitary

$UU^\dagger = U^\dagger U = I = \begin{pmatrix}1&0\\0&1\end{pmatrix}$

$c = a + bi$
$c^* = a - bi$

conjugate transpose

$\begin{pmatrix}1&2\\3&4\end{pmatrix}^\dagger = \begin{pmatrix}1^*&3^*\\2^*&4^*\end{pmatrix} = \begin{pmatrix}1&3\\2&4\end{pmatrix}$

$\begin{pmatrix}i&0\\0&-i\end{pmatrix}^\dagger = \begin{pmatrix}-i&0\\0&i\end{pmatrix}$

$\frac{1}{\sqrt2}\begin{pmatrix}1&1\\1&-1\end{pmatrix}$ = Hadamard $\boxed{H}$ valid?

Conj trans:

$\frac{1}{\sqrt2}\frac{1}{\sqrt2}\begin{pmatrix}1&1\\1&-1\end{pmatrix}\begin{pmatrix}1&1\\1&-1\end{pmatrix} = \frac12\begin{pmatrix}2&0\\0&2\end{pmatrix} = I$

$UU^\dagger = I$
$U^\dagger U = I$

$H|0\rangle =$
$= \frac{1}{\sqrt2}\begin{pmatrix}1&1\\1&-1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \frac{1}{\sqrt2}\begin{pmatrix}1\\1\end{pmatrix} = |+\rangle$

$H|1\rangle =$
$= \frac{1}{\sqrt2}\begin{pmatrix}1&1\\1&-1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = \frac{1}{\sqrt2}\begin{pmatrix}1\\-1\end{pmatrix} = |-\rangle$

3-qubit system → states? → gates?

n-qubit

000
001 8 possible comb. → 8 basis states

2^n possible comb → 2^n basis states

$H|0\rangle$

$U|\psi\rangle = |\psi'\rangle$

$2^n \times 2^n$ $\quad 2^n$

$|00\rangle$ —[U]— $\frac12\begin{pmatrix}1\\1\\1\\1\end{pmatrix}$

$\begin{array}{c} \begin{pmatrix}1\\0\\0\\0\end{pmatrix} \end{array}\begin{array}{l} \leftarrow 00\\ \leftarrow 01\\ \leftarrow 10\\ \leftarrow 11\end{array}$

$\frac12\left(\begin{array}{cc|cc}1&1&1&1\\1&-1&1&-1\\\hline 1&1&-1&-1\\1&-1&-1&1\end{array}\right)\begin{pmatrix}1\\0\\0\\0\end{pmatrix} = \frac12\begin{pmatrix}1\\1\\1\\1\end{pmatrix}$

uniform superpos:

$|+\rangle, |-\rangle$ $\quad \frac{1}{\sqrt2}\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}\frac{1}{\sqrt2}\\\frac{1}{\sqrt2}\end{pmatrix}$

$c\begin{pmatrix}1\\1\\1\\1\\1\\1\\1\\1\end{pmatrix} = \begin{pmatrix}c\\c\\c\\c\\c\\c\\c\\c\end{pmatrix}$ $\quad |c|^2 \quad \frac{1}{\sqrt8} + \frac{1}{\sqrt8} + \ldots = 1$

$\frac{1}{2\sqrt2}$

n-qubits: $\frac{1}{\sqrt{2^n}} = 2^{-\frac{n}{2}}$

$|0\rangle = \begin{pmatrix}1\\0\end{pmatrix}$ = photon is on upper track

$|1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$ = photon is on lower track

$H\begin{pmatrix}0\\1\end{pmatrix} = H|1\rangle = |-\rangle$

matrix: $H|-\rangle = \frac{1}{\sqrt2}\begin{pmatrix}1&1\\1&-1\end{pmatrix}\cdot\frac{1}{\sqrt2}\begin{pmatrix}1\\-1\end{pmatrix} = \frac12\begin{pmatrix}0\\2\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix} = |1\rangle$

linearity:

$H|-\rangle = H\left(\frac{1}{\sqrt2}|0\rangle - \frac{1}{\sqrt2}|1\rangle\right) \overset{lin}{=} \frac{1}{\sqrt2}H|0\rangle - \frac{1}{\sqrt2}H|1\rangle = \frac{1}{\sqrt2}(|+\rangle - |-\rangle) = \frac{1}{\sqrt2}\left(\frac{1}{\sqrt2}\begin{pmatrix}1\\1\end{pmatrix} - \frac{1}{\sqrt2}\begin{pmatrix}1\\-1\end{pmatrix}\right) = \frac{1}{\sqrt2}\left(\frac{1}{\sqrt2}\begin{pmatrix}0\\2\end{pmatrix}\right) = \frac12\begin{pmatrix}0\\2\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix}$

$\parallel$

$\frac{1}{\sqrt2}\left(\cancel{\frac{1}{\sqrt2}|0\rangle} + \frac{1}{\sqrt2}|1\rangle - \left(\cancel{\frac{1}{\sqrt2}|0\rangle} - \frac{1}{\sqrt2}|1\rangle\right)\right) = \frac{1}{\sqrt2}\left(\frac{2}{\sqrt2}|1\rangle\right) = |1\rangle$

$U_f|x\,y\rangle \to |x\,(y \oplus f(x))\rangle$

$U_f|1\,1\rangle \to |1 \quad 1\oplus f(1)\rangle$

$\uparrow = |0\rangle$ $\quad \nearrow = \frac{1}{\sqrt2}|0\rangle + \frac{1}{\sqrt2}|1\rangle$

$\rightarrow = |1\rangle$

$R_\theta R_\theta^\dagger = \begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix} = \begin{pmatrix}\overbrace{\cos\theta\cos\theta + \sin\theta\sin\theta}^{\cos^2\theta + \sin^2\theta = 1} & \overbrace{\cancel{\cos\theta\sin\theta - \sin\theta\cos\theta}}^{0}\\ 0 & 1\end{pmatrix} = \begin{pmatrix}1&0\\0&1\end{pmatrix}$

$R_\theta = \begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}$ $\qquad R_\theta^\dagger R_\theta = \ldots$

$R_{90} = \begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$ $\qquad R_\theta|0\rangle = \begin{pmatrix}0\\1\end{pmatrix}$

$R_\theta|1\rangle = -\begin{pmatrix}1\\0\end{pmatrix}$

$\begin{pmatrix}\alpha\\\beta\end{pmatrix} = \gamma R_{90}\begin{pmatrix}\alpha\\\beta\end{pmatrix} = \gamma\begin{pmatrix}-\beta\\\alpha\end{pmatrix}$

$\alpha = -\beta\gamma$
$\beta = \gamma\alpha$
$\alpha = -\gamma\gamma\alpha = -1\cdot\gamma^2\alpha$

$\gamma^2 = -1$
$\gamma_1 = i$
$\gamma_2 = -i$

$\gamma_1 = i \quad \alpha = 1, \beta = i$ Circular polarization

$|\varphi_1\rangle = \frac{1}{\sqrt2}\begin{pmatrix}1\\i\end{pmatrix} \leftarrow L$

$\gamma_2 = -i \quad \alpha = 1, \beta = -i$

$|\varphi_2\rangle = \frac{1}{\sqrt2}\begin{pmatrix}1\\-i\end{pmatrix} \leftarrow R$

$|light_1\rangle = |\varphi_1\rangle$
$|light_2\rangle = |\varphi_2\rangle$

$|\varphi_1\rangle = \alpha|0\rangle + \beta|1\rangle$

$P_r[|0\rangle] = |\alpha|^2$
$P_r[|1\rangle] = |\beta|^2$