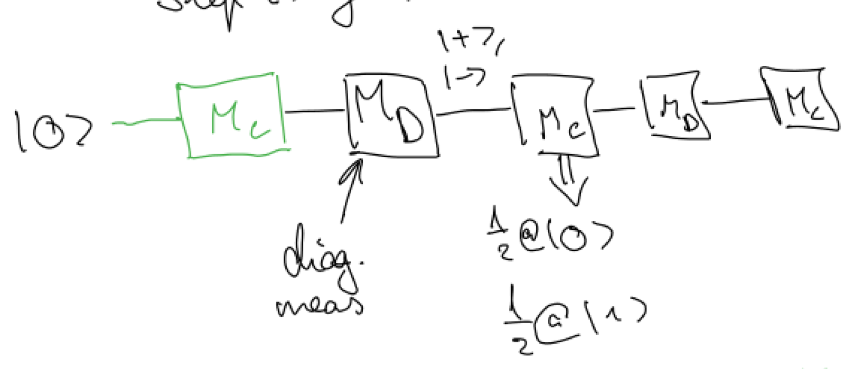
# QC lab March 3

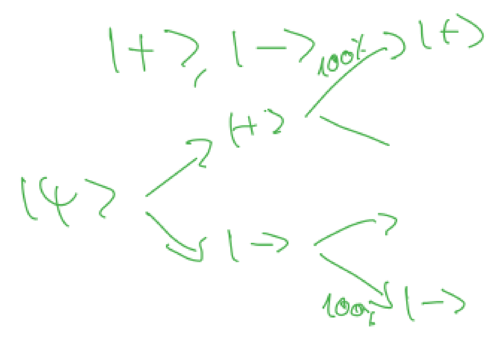
How to initialize state to $|\psi\rangle$ only using meas.

Simple example: I want $|\psi\rangle = |1\rangle$

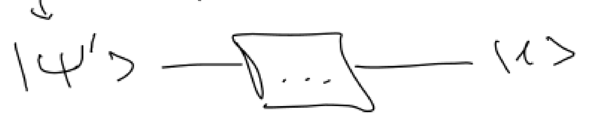
Start: $|0\rangle$

Step 1: gen. rand. values



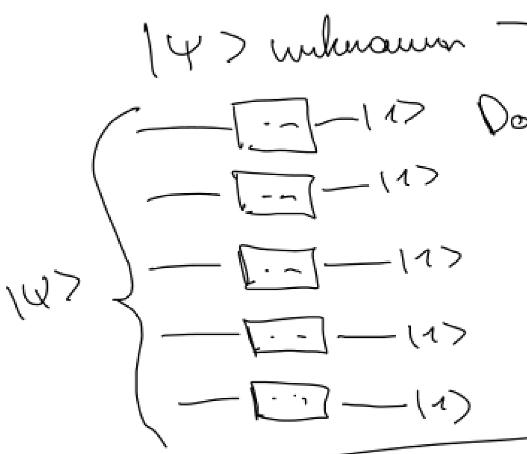




??? Repeat this until $|1\rangle$

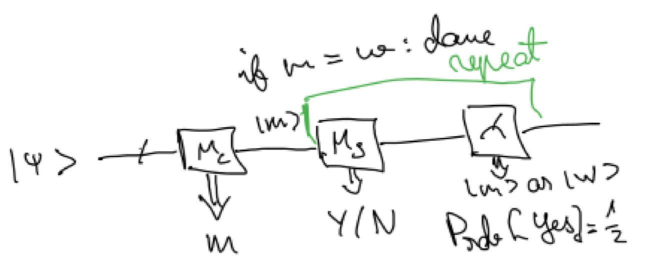



## Multiple qubits

$|\psi\rangle$ unknown $\rightarrow |11\ldots1\rangle =: w$




Do the above separately




if $|w\rangle \rightarrow$ done
$|m\rangle \rightarrow$ repeat

$\rightarrow M_{YES} = \left\{ \frac{1}{\sqrt{2}}(|m\rangle + |w\rangle), \frac{1}{\sqrt{2}}(|m\rangle - |w\rangle) \right\}$

$M_{NO} = \{ |k\rangle \mid k \notin \{w, m\} \}$

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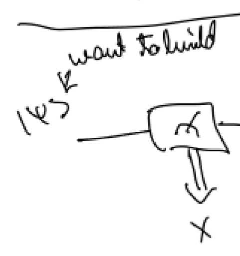



Given $D$ on $X$, build $|\psi\rangle$ s.t. comp.-measurements on $|\psi\rangle$ follow $D$

$P_\Omega[\text{measuring } |\psi\rangle \text{ gives } |x\rangle] = P_D[\text{Sampling } D \text{ gives } x]$
$\qquad = D(x)$

Simpler version:

1 bit $\quad D = $ uniform $\qquad |\psi\rangle = |+\rangle, |-\rangle$

$\frac{1}{2}$@$0 \qquad \frac{1}{2}$@$1$

$\frac{1}{4}$@$0 \qquad \frac{3}{4}$@$1 \qquad |\psi\rangle = \frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$

$p_0$@$0 \qquad p_1$@$1 \qquad |\psi\rangle = \sqrt{p_0}|0\rangle + \sqrt{p_1}|1\rangle$

$|7\rangle := |bin\,7\rangle$

n bits

$p_0 = P_D[D \text{ gives } 0] = D(0)$

$p_1 = D(1)$

$\vdots$

$p_N = D(N)$

$|\psi\rangle = \sqrt{D(0)}\,\underbrace{|0\ldots0\rangle}_{=:0} + \sqrt{D(1)}\,\underbrace{|0\ldots1\rangle}_{=:1} = \ldots$

$= \sum_{i \in \{0..N-1\}} \sqrt{D(i)}\,|i\rangle$

#qubits

$N = 2^n - 1 \quad [7]$

$|i\rangle$ were basis states

$\langle i|j\rangle \begin{cases} 1 & \text{if } i=j \\ 0 & \text{otherwise} \end{cases}$

Show it's a state:

$\||\psi\rangle\|^2 = \langle\psi|\psi\rangle = \left(\sum_i \sqrt{D(i)}|i\rangle\right)^\dagger \left(\sum_i \sqrt{D(i)}|i\rangle\right) = \sum_{i,j} \sqrt{D(i)}^* \sqrt{D(j)} \langle i|j\rangle = \sum_i D(i) \ast = 1$

$\langle\psi| = |\psi\rangle^\dagger$

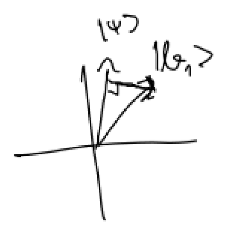
Start from $|0\rangle$, have to get $|\psi\rangle$ (this time w/ unitaries)

Want $U$: $\quad U|0\rangle = |\psi\rangle \quad$ Hint: Square $U$ is unitary iff its cols are orthonormal (orthogonal + norm 1)

$\begin{pmatrix} |\psi\rangle & ??? \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = |\psi\rangle$

Want basis with $|\psi\rangle$ in it.

Have $|\psi\rangle$, pick randomly $|b_1\rangle$, is it suitable for basis vector?




no $\rightarrow$ not ON w/ $|\psi\rangle$

$|b_1'\rangle = |b_1\rangle - \mathrm{proj}_\psi(b_1) \leftarrow$ not length 1

basis vector $|b_1''\rangle = \frac{|b_1'\rangle}{\||b_1'\rangle\|}$

Gram-Schmidt

Repeat to fill cols of $U$

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$U$ is unitary if:

1) $U^\dagger U = I$
2) $UU^\dagger = I$
3) cols are ONB
4) rows are ONB
5) Preserves length
6) $U^\dagger$ is unitary

$1 \Rightarrow 2$

$U^\dagger U = I \Rightarrow UU^\dagger = I$ ✓

$UU^\dagger = U\underbrace{U^\dagger U}_{=I}\underbrace{U^{-1}}_{=I} = UU^{-1} = I$

$2 \Rightarrow 1$

$UU^\dagger = I \Rightarrow U^\dagger U = I$

$3 \Rightarrow 1$

$U$ cols are ON $\Rightarrow U^\dagger U = I$

$(U^\dagger U)_{k,l} = \langle U_k, U_l \rangle = \begin{cases} 1 & \text{if } k=l \\ 0 & \text{otherwise} \end{cases}$

$1 \Rightarrow 3$

$U^\dagger U = I \Rightarrow$ cols are ONB

$U|i\rangle, U|j\rangle$ are cols

$\langle U|i\rangle, U|j\rangle\rangle = (U|i\rangle)^\dagger (U|j\rangle) = \langle i|U^\dagger U|j\rangle = \langle i|j\rangle \begin{cases} 1 & \text{if } i=j \\ 0 & \text{otherwise} \end{cases}$