QC lab March 9th

$\otimes$ tensor: $A = \begin{pmatrix} A_{11} & A_{12} & \cdots \\ A_{21} & & \end{pmatrix} \otimes B = \begin{pmatrix} A_{11}\cdot B & A_{12}B & \cdots \\ A_{21}\cdot B & \ddots & \\ \vdots & & \ddots \end{pmatrix}$

$|00\rangle = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}$ $\quad |0\rangle\otimes|0\rangle = \begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\cdot B\\0\cdot B\end{pmatrix} = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}$

$|00\rangle$ — H, H $\quad U = \underline{H\otimes H}$

$(A\otimes B)|\psi\rangle\otimes|\varphi\rangle = A|\psi\rangle\otimes B|\varphi\rangle$

$|\psi\rangle$ — A

$|\varphi\rangle$ — B

$(A+B)\otimes C = A\otimes C + B\otimes C$

$(H\otimes H)(|0\rangle\otimes|0\rangle) = H|0\rangle\otimes H|0\rangle =$

$= |+\rangle\otimes|+\rangle = \left(\frac{1}{\sqrt2}|0\rangle + \frac{1}{\sqrt2}|1\rangle\right)\otimes\left(\frac{1}{\sqrt2}|0\rangle + \frac{1}{\sqrt2}|1\rangle\right) = \frac12|00\rangle + \frac12|01\rangle + \frac12|10\rangle + \frac12|11\rangle$

$\beta_{00} = \frac{1}{\sqrt2}|00\rangle + \frac{1}{\sqrt2}|11\rangle$ $\quad \beta_{00}$ — M

$|+\rangle$ — M_2 $\quad$ "Are you in $|\psi\rangle$?" $\quad |\psi\rangle\langle\psi|$

$P_1 \quad P_2 \quad \ldots$ orth. span the space $\sum_i P_i = I_n$

$|0\rangle \quad |1\rangle$

$\begin{pmatrix}1\\0\end{pmatrix}(1\ 0) = \begin{pmatrix}1&0\\0&0\end{pmatrix}$ $\quad \begin{pmatrix}0\\1\end{pmatrix}(0\ 1) = \begin{pmatrix}0&0\\0&1\end{pmatrix}$

$M_0 = I\otimes|0\rangle\langle0|$

$M_1 = I\otimes|1\rangle\langle1|$

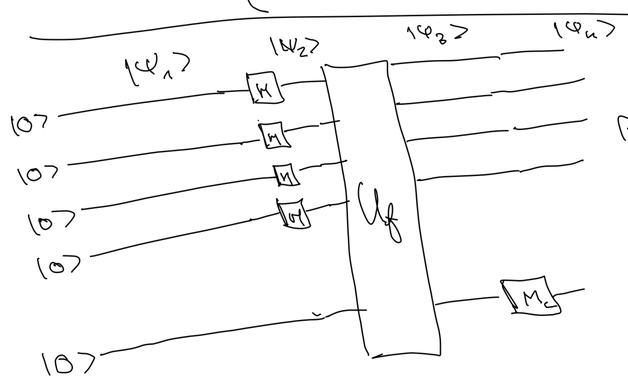
$M_0\beta_{00} = M_0\left(\frac{1}{\sqrt2}|00\rangle + \frac{1}{\sqrt2}|11\rangle\right) = \frac{1}{\sqrt2}M_0|00\rangle + \frac{1}{\sqrt2}M_0|11\rangle = \frac{1}{\sqrt2}|00\rangle$ ✗

$(I\otimes|0\rangle\langle0|)(|0\rangle\otimes|0\rangle) = |0\rangle\otimes|0\rangle\overbrace{\langle0|0\rangle}^{1} = |0\rangle\otimes|0\rangle$

$(I\otimes|0\rangle\langle0|)(|1\rangle\otimes|1\rangle) = |1\rangle\otimes|0\rangle\overbrace{\langle0|1\rangle}^{0}$

$Pr[M_0] = \left\|\frac{1}{\sqrt2}|00\rangle\right\|^2 = \frac12$

$PMS[M_0] = \frac{\frac{1}{\sqrt2}|00\rangle}{\left\|\frac{1}{\sqrt2}|00\rangle\right\|} = |00\rangle$



State here

$U_f|a,b\rangle = |a, b\oplus f(a)\rangle$ $\quad |a\rangle\otimes|b\rangle$

$f = \text{parity}(a)$

parity of bitstring of a: 1 if odd, 0 if even

$|\psi_1\rangle = |0\rangle\otimes|0\rangle\otimes|0\rangle\otimes|0\rangle\otimes|0\rangle$

$|\psi_2\rangle = (H\otimes H\otimes H\otimes H\otimes I)|\psi_1\rangle = |+\rangle\otimes|+\rangle\otimes|+\rangle\otimes|+\rangle\otimes|0\rangle =$

$= \frac{1}{\sqrt2}(|0\rangle+|1\rangle)\otimes\ldots\otimes\ldots\otimes\ldots\otimes|0\rangle = 2^{-2}\sum_{i\in\{0000\ldots1111\}}|i\rangle\otimes|0\rangle$

$|\psi_3\rangle = U|\psi_2\rangle = U\left(2^{-2}\sum_{i\in\{0000\ldots1111\}}|i\rangle\otimes|0\rangle\right) = 2^{-2}\sum_i U(|i\rangle\otimes|0\rangle) = 2^{-2}\left(\sum_{i \text{ is "even"}}|i\rangle\otimes|0\rangle + \sum_{j \text{ is "odd"}}|j\rangle\otimes|1\rangle\right)$

$P_0 = I^{\otimes4}\otimes|0\rangle\langle0|$

$P_0(|j\rangle\otimes|1\rangle) = |j\rangle\otimes|0\rangle\overbrace{\langle0|1\rangle}^{0} = |j\rangle\otimes|0\rangle\cdot0$

Proj. for measurement

$P_0 = I^{\otimes4}\otimes|0\rangle\langle0|$

$P_1 = I^{\otimes4}\otimes|1\rangle\langle1|$

Apply P_0 to $|\psi_3\rangle$:

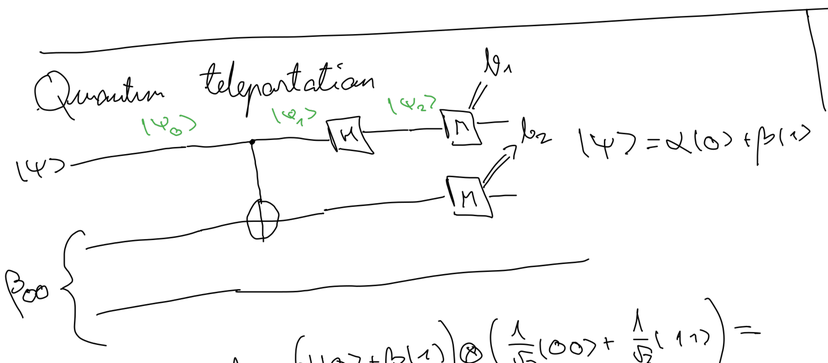
$P_0|\psi_3\rangle = P_0\,2^{-2}\left(\sum_{i \text{ is "even"}}|i\rangle\otimes|0\rangle + \sum_{j \text{ is "odd"}}|j\rangle\otimes|1\rangle\right) = 2^{-2}\left(\sum_{i \text{ is even}}P_0|i\rangle\otimes|0\rangle + \sum_{j \text{ is odd}}P_0|j\rangle\otimes|1\rangle\right) = 2^{-2}\left(\sum_{i \text{ is even}}|i\rangle\otimes|0\rangle\right)$

$Pr \text{ of } 0 = \|P_0|\psi_3\rangle\|^2$

$Pr[M_0] = \frac{\#\text{ even vectors}}{\text{total}}$

$Pr[M_1] = \frac{\#\text{ odd bit vectors}}{\text{total}}$

Quantum teleportation

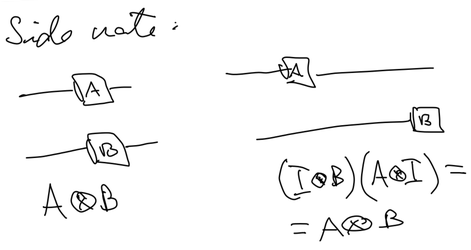


$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

$|\psi_0\rangle = |\psi\rangle\otimes\beta_{00} = (\alpha|0\rangle+\beta|1\rangle)\otimes\left(\frac{1}{\sqrt2}|00\rangle + \frac{1}{\sqrt2}|11\rangle\right) =$

$= \frac{\alpha}{\sqrt2}|000\rangle + \frac{\alpha}{\sqrt2}|011\rangle + \frac{\beta}{\sqrt2}|100\rangle + \frac{\beta}{\sqrt2}|111\rangle$

Side note:



$A\otimes B$ $\quad (I\otimes B)(A\otimes I) = A\otimes B$

$|\psi_1\rangle = (CNOT\otimes I)|\psi_0\rangle = \frac{\alpha}{\sqrt2}|000\rangle + \frac{\alpha}{\sqrt2}|011\rangle + \frac{\beta}{\sqrt2}|110\rangle + \frac{\beta}{\sqrt2}|101\rangle$

$|0\rangle\otimes|00\rangle$

$|\psi_2\rangle = (H\otimes I\otimes I)|\psi_1\rangle = \frac{\alpha}{\sqrt2}\left(\frac{1}{\sqrt2}|0\rangle + \frac{1}{\sqrt2}|1\rangle\right)\otimes|00\rangle + \frac{\alpha}{\sqrt2}\left(\frac{1}{\sqrt2}|0\rangle + \frac{1}{\sqrt2}|1\rangle\right)\otimes|11\rangle + \frac{\beta}{\sqrt2}\left(\frac{1}{\sqrt2}|0\rangle - \frac{1}{\sqrt2}|1\rangle\right)\otimes|10\rangle + \frac{\beta}{\sqrt2}\left(\frac{1}{\sqrt2}|0\rangle - \frac{1}{\sqrt2}|1\rangle\right)\otimes|01\rangle =$

$\alpha|00\rangle\otimes|\psi\rangle$

$= \frac{\alpha}{2}|000\rangle + \frac{\alpha}{2}|100\rangle + \frac{\alpha}{2}|011\rangle + \frac{\alpha}{2}|111\rangle + \frac{\beta}{2}|010\rangle - \frac{\beta}{2}|110\rangle + \frac{\beta}{2}|001\rangle - \frac{\beta}{2}|101\rangle$

$= \frac12\Big(|00\rangle\otimes\underbrace{(\alpha|0\rangle+\beta|1\rangle)}_{|\psi\rangle} + |01\rangle\otimes(\alpha|1\rangle+\beta|0\rangle) + |10\rangle\otimes(\alpha|0\rangle-\beta|1\rangle) + |11\rangle\otimes(\alpha|1\rangle-\beta|0\rangle)\Big)$

$Z|0\rangle \Rightarrow |0\rangle$

$Z|1\rangle \Rightarrow -1|1\rangle$

b_1	b_2	
0	0	ihave $|\psi\rangle$ on third qubit
1	0	$I\otimes I\otimes Z$ gives $|\psi\rangle$
0	1	$I\otimes I\otimes X$ gives $|\psi\rangle$
1	1	need to do both