Graph Isomorphism

General

Given two graphs, G and H, we say that G is isomorphic to H if there exists a one-to-one correspondence between the vertices of G and H such that two vertices are adjacent in G if and only if their corresponding vertices are adjacent in H.

**Example:****

Let G be a graph with vertices {A, B, C, D} and edges {AB, BC, CD}. Let H be a graph with vertices {1, 2, 3, 4} and edges {1, 2, 3}. Then G is isomorphic to H if we can find a one-to-one correspondence between the vertices of G and H that preserves adjacency.

A possible correspondence is:

- A ↔ 1
- B ↔ 2
- C ↔ 3
- D ↔ 4

In this correspondence, the adjacency relations are preserved:

G: AB, BC, CD
H: 1, 2, 3

Therefore, G is isomorphic to H.

**Exercise:****

Show that the graphs G and H are not isomorphic.

**Solution:****

Consider the degree of each vertex in G and H. The degree of A is 2, the degree of B is 2, the degree of C is 2, and the degree of D is 2 in G. The degree of 1 is 2, the degree of 2 is 2, the degree of 3 is 2, and the degree of 4 is 2 in H. Since the degree of each vertex is the same in both graphs, we cannot find a one-to-one correspondence that preserves adjacency. Therefore, G is not isomorphic to H.