1 Regev’s cryptosystem

In Regev’s cryptosystem, we have an error term $e$ that is initialized according to a distribution $\chi$. In this homework, we investigate what happens, say due to a programmer error, $e$ is not properly randomized.

(a) Knowlets: Regev, CompLWE
Time: 
Difficulty: 

We have a faulty implementation of Regev’s cryptosystem where $e = (0, \ldots, 0)$ always. The adversary gets the public-key $(A, b)$ and a ciphertext $(c_1, c_2)$. How can the adversary compute the plaintext? (Describe the computation steps performed by the adversary.)

Hint: If in doubt, first try to figure out how to solve the computational LWE problem (i.e., find $s$) when $e = 0$ always.

(b) Knowlets: Regev, CompLWE
Time: 
Difficulty: 

Now we have a slightly better implementation. $e$ now indeed contains some noise, but too little. In fact, it turns out that with probability close to 1, only one component $e_i \neq 0$. (That is, for all $j \neq i$, $e_j = 0$.) Show that this is too little noise by giving an attack. (Given public key and ciphertext find the plaintext. Describe the computation steps performed by the adversary.)

(c) Knowlets: Regev
Time: 
Difficulty: 

Now we have a different randomness failure. $e$ is chosen properly, but $A = 0$. How to attack? (Given public key and ciphertext find the plaintext. Describe the computation steps performed by the adversary.)

(d) Knowlets: Regev
Time: 
Difficulty: 

Consider the following variant of Regev’s scheme:
• Encryption. To encrypt $\mu \in \mathbb{Z}_q$, pick $x \overset{\$}{\leftarrow} \{0,1\}^m$. Let $c_1 := A^T x$ and $c_2 := x \cdot b + \mu$ (all calculated in $\mathbb{Z}_q$).

That is, we have optimized the scheme by allowing messages in $\mathbb{Z}_q$ (i.e., not limited to a single bit). This is much more efficient. What is the problem with this change?

And now something completely different: Given a ciphertext $(c_1, c_2)$ that is the encryption of some unknown $\mu \in \{0,1\}$, how to compute a ciphertext $(c'_1, c'_2)$ that decrypts to $1 - \mu$ (with high probability)?

**Note:** You do not need to prove that your solution is correct, it is enough to specify the algorithm.

**Note:** What you are showing here is that Regev’s cryptosystem is malleable.

<table>
<thead>
<tr>
<th>Knowlets:</th>
<th>Regev</th>
<th>ProblemID: RegevMall</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time:</td>
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<td>Difficulty:</td>
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