WiM — a simple abstract machine for logical languages

The language Proll

We consider a small logic programming language Proll ("Prolog-light").

Compared with Prolog, we do not treat:

- arithetic operations;
- the cut operator;
- self-modification of programs using `assert` and `retract`.

The language Proll

A program p has the following syntax:

$$
\begin{array}{lcl}
t & ::= & a \mid X \mid \_ \mid f(t_1, \ldots, t_n) \\
g & ::= & p(t_1, \ldots, t_k) \mid X = t \\
c & ::= & p(X_1, \ldots, X_k) \leftarrow g_1, \ldots, g_r \\
p & ::= & c_1 \ \ldots \ c_m \ ?g
\end{array}
$$

- A *term* t is either an atom (ie. constant), a variable, an anonymous variable, or a constructor application.
- A *goal* g is either literal, ie. a predicate call, or a unification.
- A *clause* c has a *head* $p(X_1, \ldots, X_k)$ and a *body* (ie. a sequence of goals).
- A *program* consits of sequence of clauses together with a *query* (ie. a single top-level goal).

The language Proll

Example:

$$\begin{array}{lcl}
\texttt{bigger}(X,Y) & \leftarrow & X = \mathit{elephant},\ Y = \mathit{horse} \\
\texttt{bigger}(X,Y) & \leftarrow & X = \mathit{horse},\ Y = \mathit{donkey} \\
\texttt{bigger}(X,Y) & \leftarrow & X = \mathit{donkey},\ Y = \mathit{dog} \\
\texttt{bigger}(X,Y) & \leftarrow & X = \mathit{donkey},\ Y = \mathit{monkey} \\
\texttt{is_bigger}(X,Y) & \leftarrow & \texttt{bigger}(X,Y) \\
\texttt{is_bigger}(X,Y) & \leftarrow & \texttt{bigger}(X,Z),\ \texttt{is_bigger}(Z,Y) \\
\texttt{?is_bigger}(\mathit{elephant}, \mathit{dog}) & &
\end{array}$$

The language Proll

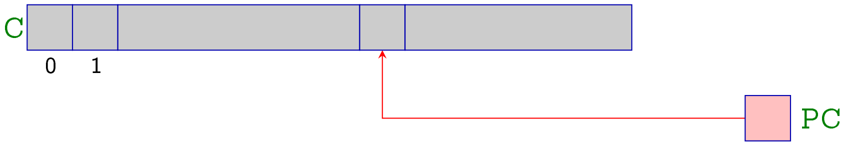
Example:

$$
\begin{array}{lcl}
\mathrm{app}(X, Y, Z) & \leftarrow & X = [\,],\ Y = Z \\
\mathrm{app}(X, Y, Z) & \leftarrow & X = [H \mid X'],\ Z = [H \mid Z'],\ \mathrm{app}(X', Y, Z') \\
?\mathrm{app}(X, [Y, c], [a, b, Z]) & &
\end{array}
$$

- $[\,]$ the atom denoting an empty list;
- $[H \mid Z]$ a binary list constructor application;
- $[a, b, Z]$ is a shorthand of $[a \mid [b \mid [Z \mid [\,]]]]$.

WiM architecture

Code:

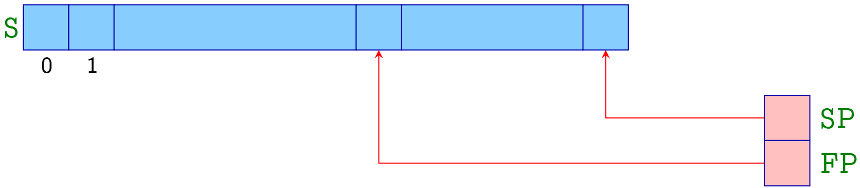


C	=	Code-store — memory area for a program code; each cell contains a single AM instruction.
PC	=	Program Counter — register containing an address of the instruction to be executed *next*.

Initially, PC contains the address 0; ie. C[0] contains the first instruction of the program.

WiM architecture

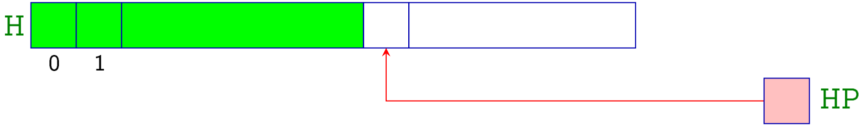
Stack:



S	=	Stack — each cell contains a primitive value or an address;
SP	=	Stack-Pointer — points to top of the stack;
FP	=	Frame-Pointer — points to the currently active frame.

WiM architecture

Heap:

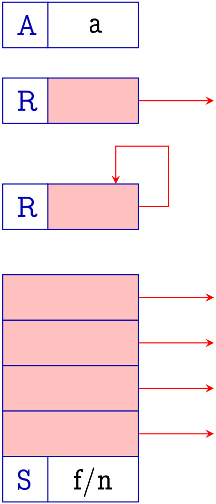


H = Heap — memory area for dynamically allocated data;
HP = Heap-Pointer — points to the first free cell.

- The instruction **new** creates a new object in the heap.
- Objects are tagged with their types (like in MaMa).

WiM architecture

Heap may contain following objects:



Atom	1 cell
Variable	1 cell
Unbound Variable	1 cell
Structure	n+1 cells

Construction of Terms

Before parameters are passed to goals, the corresponding terms are constructed in the heap.

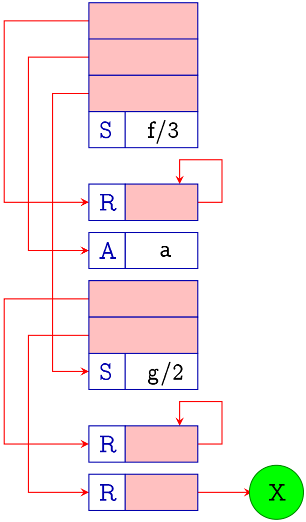
The address environment ρ binds each clause variable X with its address in the stack (relative of FP).

Construction of terms is performed by function $\text{code}_A\ t\ \rho$, which:

- creates a tree representation of the term t in the heap ;
- returns a pointer to it on top of the stack.

Construction of Terms

Example: Represention of the term $t \equiv f(g(X, Y), a, Z)$, where X is an initialized variable, and Y and Z are not yet initialized.



Construction of Terms

$$
\begin{array}{lcl}
\text{code}_A\ a\ \rho & = & \texttt{putatom a} \\
\text{code}_A\ X\ \rho & = & \texttt{putvar}\ (\rho\ X) \\
\text{code}_A\ \bar{X}\ \rho & = & \texttt{putref}\ (\rho\ X) \\
\text{code}_A\ \_\ \rho & = & \texttt{putanon}
\end{array}
$$

$$
\begin{array}{l}
\text{code}_A\ f(t_1, \ldots, t_n)\ \rho \quad = \\
\qquad \text{code}_A\ t_1\ \rho \\
\qquad \ldots \\
\qquad \text{code}_A\ t_n\ \rho \\
\qquad \texttt{putstruct f/n}
\end{array}
$$

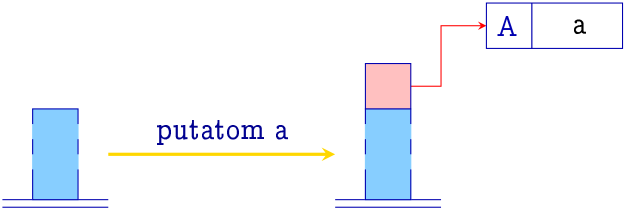
where X is an uninitialized and $\bar{X}$ is an initialized variable.

Example: let $t \equiv f(g(\bar{X}, Y), a, Z)$ and $\rho = \{X \mapsto 1, Y \mapsto 2, Z \mapsto 3\}$, then

$\text{code}_A\ t\ \rho$ emits the code:

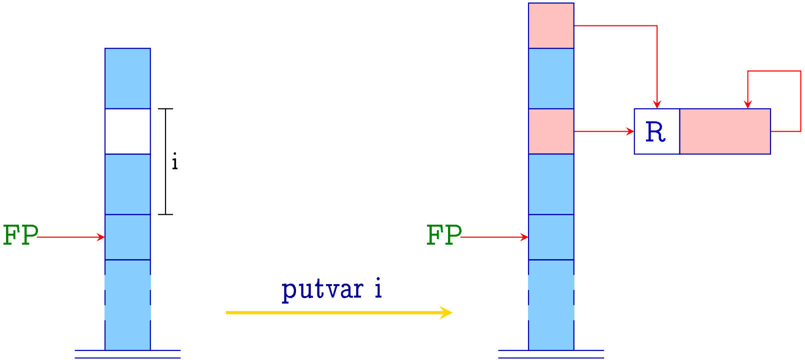
```
putref 1          putatom a
putvar 2          putvar 3
putstruct g/2     putstruct f/3
```

Construction of Terms



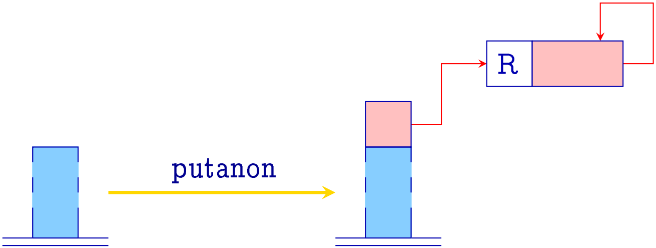

```
SP++;
S[SP] = new (A,a);
```

Construction of Terms



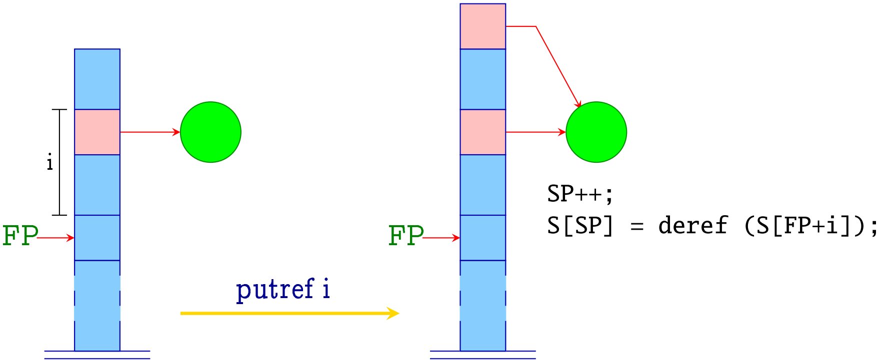

```
SP++;
S[SP] = new (R,HP);
S[FP+i] = S[SP];
```

Construction of Terms




```
SP++;
S[SP] = new (R,HP);
```

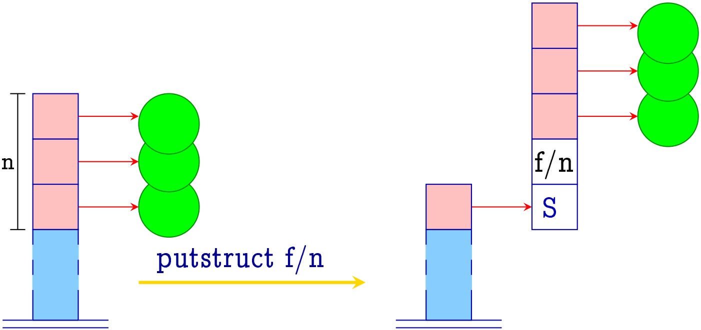
Construction of Terms



The auxiliary function deref contracts chains of references:

```
ref deref (ref v) {
  if (H[v] = (R,w) && v ≠ w) return deref(w);
  else return v;
}
```

Construction of Terms




```
v = new (S,f,n);
SP = SP - n + 1;
for (i=1; i≤n; i++)
  H[v+i] = SP[SP+i-1];
S[SP] = v;
```

Construction of Terms

Remarks:

- The instruction `putref i` not only copies a reference from `S[FP+i]`, but also derefernces it as much as possible.
- During term constrcution references always point to smaller heap addresses. Even though, this is also case in many other situations, it is not guaranteed in general.

Translation of Goals

- Goals correspond to procedure calls.
- Their translation is performed by the function code_G.
- First create a stack frame.
- Then construct the actual parameters in the heap
- ... and store references to these into the stack frame.
- Finally, jump to the code of the predicate.

Translation of Goals

$$
\begin{array}{rcll}
\text{code}_G\ p(t_1,\ldots,t_k)\ \rho & = & & \texttt{mark A} \\
 & & & \text{code}_A\ t_1\ \rho \\
 & & & \ldots \\
 & & & \text{code}_A\ t_k\ \rho \\
 & & & \texttt{call p/k} \\
 & & \texttt{A:} & \ldots
\end{array}
$$

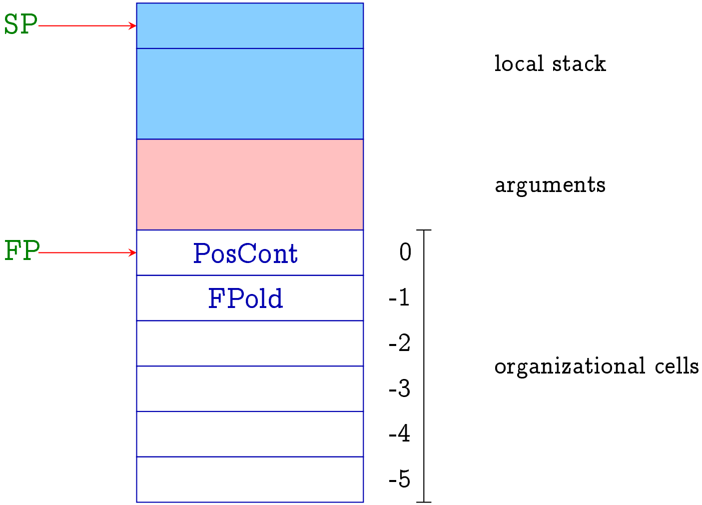
Example: let $g \equiv p(a, X, g(\bar{X}, Y))$ and $\rho = \{X \mapsto 1, Y \mapsto 2\}$,

then $\text{code}_G\ g\ \rho$ emits the code:

```
mark A          putref 1            call p/3
putatom a       putvar 2        A:  ...
putvar 1        putstruct g/2
```

Translation of Goals

Structure of a frame:

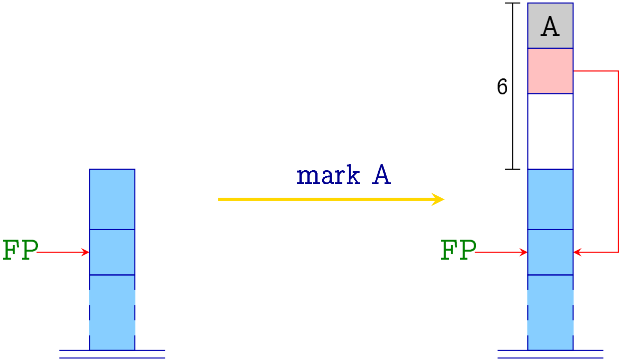


Translation of Goals

Remarks:

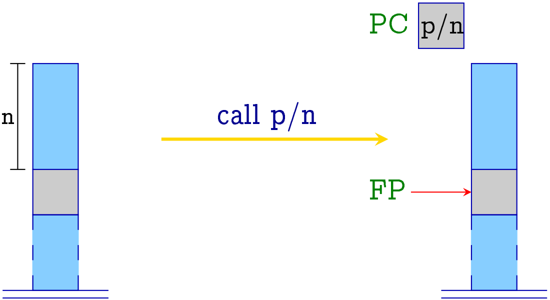
- The *positive continuation* address `PosCont` records where to continue after successful treatment of the goal.
- Additional organizational cells are necessary for *backtracking*.

Translation of Goals




```
SP = SP + 6;
S[SP] = A;
S[SP-1] = FP;
```

Translation of Goals




```
FP = SP - n;
PC = p/n;
```

Unification

- We denote occurences of a variable X by $\tilde{X}$.
- It will be translated differently depending whether it's initialized or not.
- We introduce the macro put $\tilde{X}$ ρ:

$$\begin{aligned}
\text{put } X \ \rho &= \texttt{putvar}\ (\rho X) \\
\text{put } \bar{X} \ \rho &= \texttt{putref}\ (\rho X) \\
\text{put } _ \ \rho &= \texttt{putanon}
\end{aligned}$$

Unification

Translation of the unification $\tilde{X} = t$:

- push a reference to X onto the stack;
- construct the term t in the heap;
- introduce a new instruction which implements the unification.

$$
\begin{array}{rcl}
\mathrm{code}_G\ (\tilde{X} = t)\ \rho & = & \mathrm{put}\ \tilde{X}\ \rho \\
 & & \mathrm{code}_A\ t\ \rho \\
 & & \mathtt{unify}
\end{array}
$$

Unification

Example: consider the equation

$$\bar{U} = f(g(\bar{X}, Y), a, Z)$$

Then, given an address environment

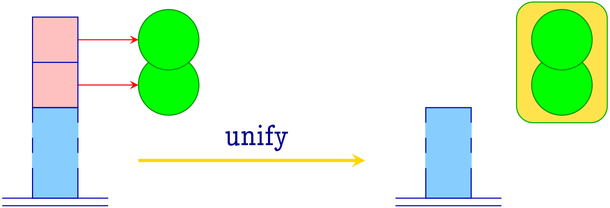
$$\rho = \{X \mapsto 1,\ Y \mapsto 2,\ Z \mapsto 3,\ U \mapsto 4\}$$

the following code is generated:

```
putref 4
putref 1
putvar 2
putstruct g/2
putatom a
putvar 3
putstruct f/3
unify
```

Unification

Instruction `unify` applies the run-time function `unify()` to the topmost two references:




```
unify (S[SP-1], S[SP-2]);
SP = SP - 2;
```

Unification

Function `unify()`

- ... takes two heap addresses. For each call we guarantee that these are *maximally dereferenced*.
- ... checks whether the two adresses are already *identical*. In that case does nothing and the unification succeeded.
- ... binds *younger variables* (larger addresses) to *older variables* (smaler addresses).
- ... when binding a variable to a term, checks whether the variable occurs inside the term (*occur-check*).
- ... records newly created bindings.
- ... may fail, in which case initiates *backtracking*.

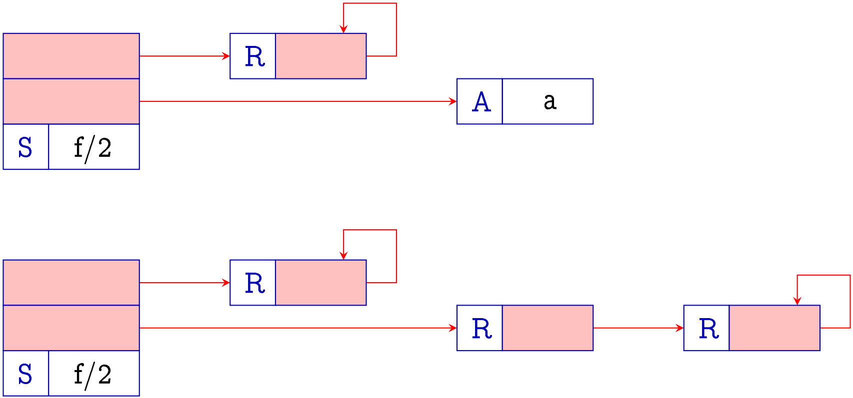
Unification

```
bool unify (ref u, ref v) {
  if (u == v) return true;
  if (H[u] == (R,_)) {
    if (H[v] == (R,_)) {
      if (u > v) {
        H[u] = (R,v); trail(u); return true;
      } else {
        H[v] = (R,u); trail(v); return true;
      }
    } else if (check (u,v)) {
      H[u] = (R,v); trail(u); return true;
    } else { backtrack(); return false; }
  }
...
```

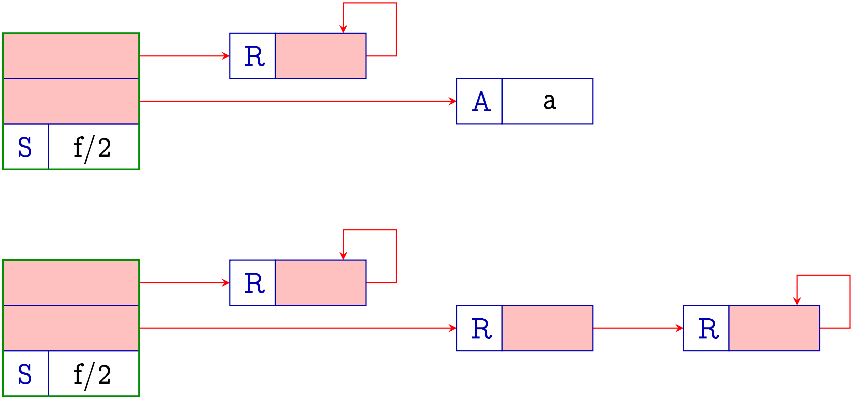
Unification

```
...
  if (H[v] == (R,_)) {
    if (check (v,u)) {
      H[v] = (R,u); trail(v); return true;
    } else { backtrack(); return false; }
  }
  if (H[u] == (A,a) && H[v] == (A,a)) return true;
  if (H[u] == (S,f/n) && H[v] == (S,f/n)) {
    for (int i=1; i<=n; i++)
      if (!unify (deref(H[u+i]), deref(H[v+i])))
          return false;
    return true;
  }
  backtrack(); return false;
}
```

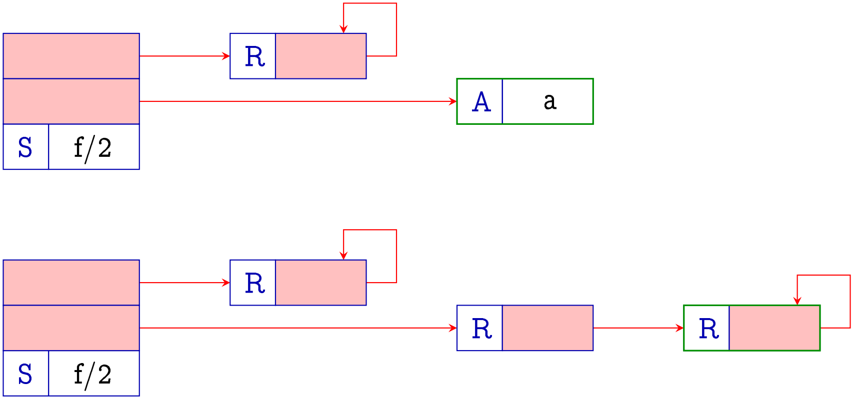
Unification



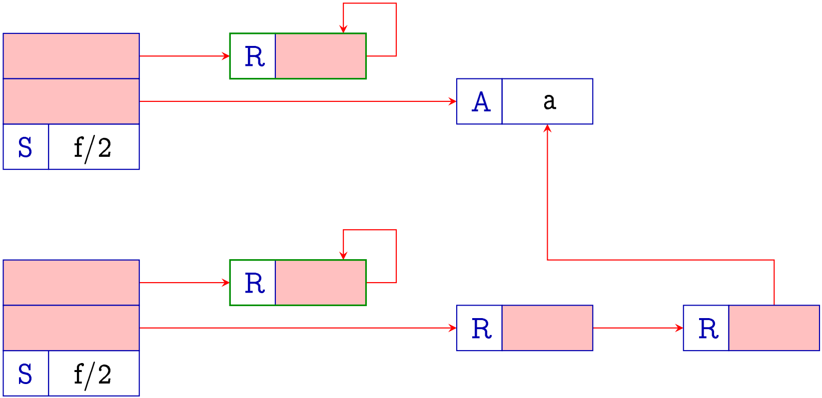
Unification



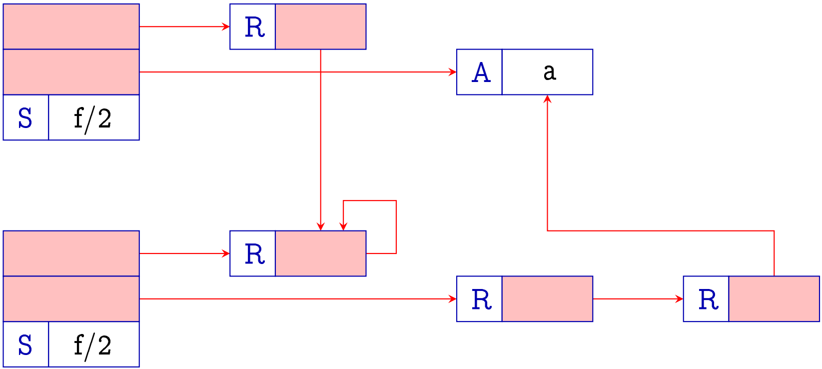
Unification



Unification



Unification



Unification

- The function `trail()` records new bindings.
- The function `backtrack()` initiates backtracking.
- The function `check()` performs the occur-check; ie. tests whether a variable (its first argument) occurs inside a term (its second argument).
- Often, this check is skipped:

```
bool check (ref u, ref v) {
    return true;
}
```

Unification

Otherwise, we could implement `check()` as follows:

```
bool check (ref u, ref v) {
  if (u == v) return false;
  if (H[v] == (S,f/n))
    for (int i=1; i<=n; i++)
      if (!check (u, deref (H[v+i])))
        return false;
  return true;
}
```

Unification

- The translation of an equation $\tilde{X} = t$ is very simple,
- but all the objects constructed to represent t which have corresponding matching object reachable from X becomes immediately *garbage*.
- Idea:
  - Push a reference to the run-time binding of $\tilde{X}$ onto the stack.
  - Avoid construction of subterms of t as long as possible.
  - Instead, translate each node of t into an instruction which performs the unification with this node!

$$\begin{array}{rcl} \text{code}_G\ (\tilde{X} = t)\ \rho & = & \text{put}\ \tilde{X}\ \rho \\ & & \text{code}_U\ t\ \rho \end{array}$$

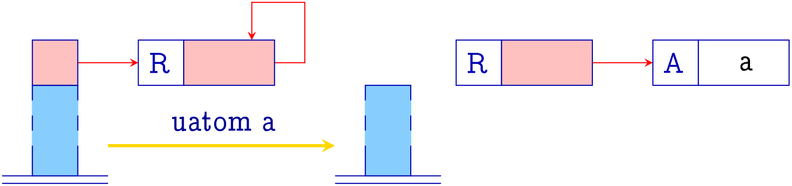
Unification

Unification of atoms and variables:

$$
\begin{array}{lcl}
\mathrm{code}_U\ a\ \rho & = & \texttt{uatom a} \\
\mathrm{code}_U\ X\ \rho & = & \texttt{uvar}\ (\rho X) \\
\mathrm{code}_U\ \bar{X}\ \rho & = & \texttt{uref}\ (\rho X) \\
\mathrm{code}_U\ \_\ \rho & = & \texttt{pop}
\end{array}
$$

Unification

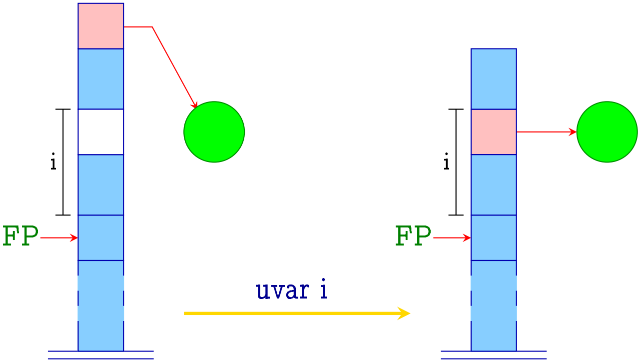
Instruction `uatom a` implements the unification with an atom:




```
v = S[SP]; SP--;
switch (H[v]) {
  case (A,a):  break;
  case (R,_):  H[v] = (R, new(A,a));
               trail(v); break;
  default:     backtrack();
}
```

Unification

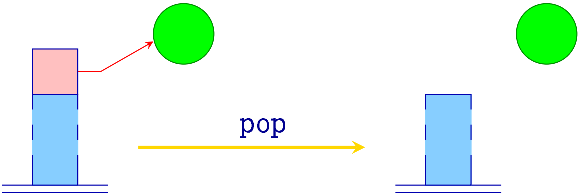
Instruction `uvar i` implements the unification with an uninitialized variable:




```
S[FP+i] = S[SP];
SP--;
```

Unification

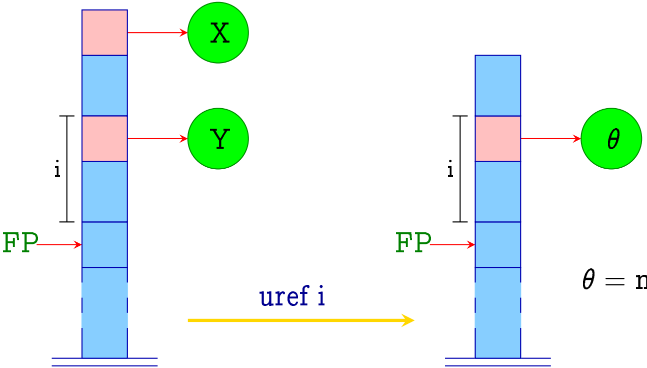
Instruction pop implements the unification with an anonymous variable:




```
SP--;
```

Unification

Instruction `uref i` implements the unification with an initialized variable:



$$\theta = \text{mgu}(X, Y)$$

```
unify (S[SP], deref(S[FP+i]));
SP--;
```

The only place, where the run-time function `unify()` is called!

Unification

Unification of constructor applications:

- The unification code performs a *pre-order* traversal over t.
- First it checks whether the root node is unifiable.
- If both terms have the same topmost constructor, then recursively checks for subterms.
- In the case of an uninitialized variable switches from checking to building.

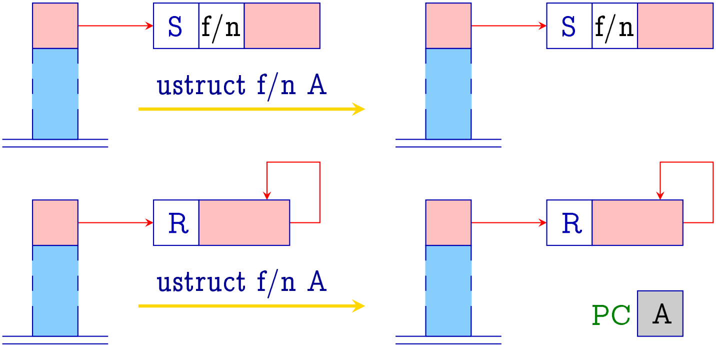
Unification

Unification of constructor applications:

$\text{code}_U\ (f(t_1, \ldots, t_n))\ \rho \quad =$

```
ustruct f/n A          up B
son 1               A: check ivars(f(t1,...,tn)) ρ
codeU t1 ρ             codeA (f(t1,...,tn)) ρ
...                    bind
son n               B: ...
codeU tn ρ
```

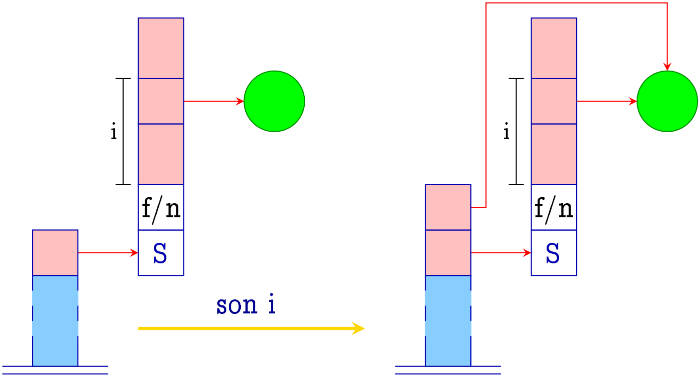
Unification




```
switch (H[S[SP]]) {
  case (S,f/n):  break;
  case (R,_):    PC = A; break;
  default:       backtrack();
}
```

Unification

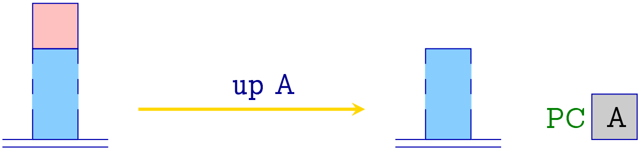
Instruction `son i` pushes the reference of the i-th subterm onto the stack:




```
S[SP+1] = deref (H[S[SP]]+i);
SP++;
```

Unification

Instruction `up A` pops a reference from the stack and jumps to the continuation address:




```
SP--;
PC = A;
```

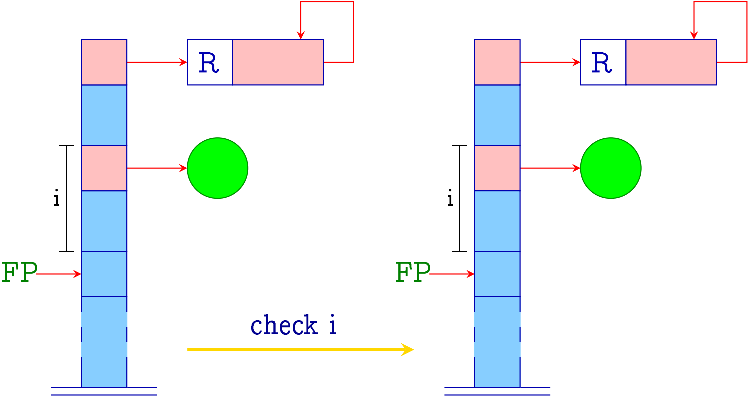
Unification

- In the case of an uninitialized variable we need to switch from checking to building.
- Before constructing the new term we need to exclude that it contains the variable on top of the stack:
  - the function ivars(t) returns the set of initialized variables of t;
  - the macro check $\{Y_1, \ldots, Y_d\}\ \rho$ generates the necessary tests:

$$
\begin{array}{rcl}
\text{check } \{Y_1, \ldots, Y_d\}\ \rho & = & \mathtt{check}\ (\rho\ Y_1) \\
 & & \ldots \\
 & & \mathtt{check}\ (\rho\ Y_d)
\end{array}
$$

Unification

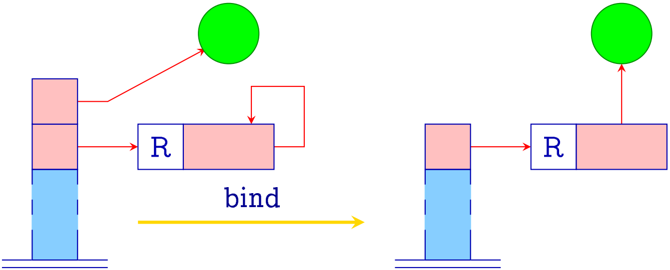
Instruction check i tests whether the (uninitialized) variable on top of the stack occurs inside the term bound to the i-th variable:




```
if (!check (S[SP], deref(S[FP+i])))
    backtrack();
```

Unification

Instruction bind binds the (uninitialized) variable to the constructed term:




```
H[S[SP-1]] = (R, S[SP]);
trail (S[SP-1]);
SP = SP - 2;
```

Unification

Example: Let $t \equiv f(g(\bar{X}, Y), a, Z)$ with environment $\rho = \{X \mapsto 1,\ Y \mapsto 2,\ Z \mapsto 3\}$. Then $\texttt{code}_U\ t\ \rho$ generates the code:

```
    ustruct f/3 A₁          putref 1              A₁: check 1
    son 1                   putvar 2                  putref 1
    ustruct g/2 A₂          putstruct g/2             putvar 2
    son 1                   bind                      putstruct g/2
    uref 1              B₂: son 2                     putatom a
    son 2                   uatom a                   putvar 3
    uvar 2                  son 3                     putstruct f/3
    up B₂                   uvar 3                    bind
A₂: check 1                 up B₁                 B₁: ...
```

Clauses

The code for clauses will:

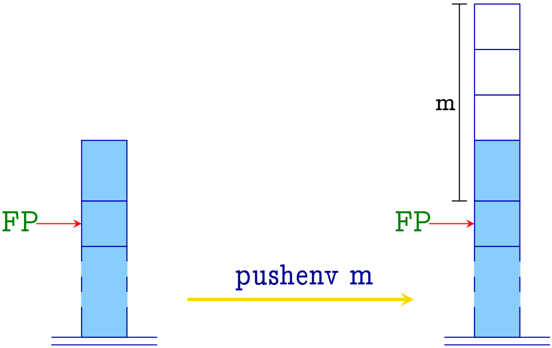
- allocate stack space for locals;
- evaluate the body;
- free the stack frame (if possible).

We denote local variables by $\{X_1, \ldots, X_m\}$, where the first k ones are formal parameters.

$$
\begin{array}{rcl}
\mathrm{code}_C\ (p(X_1, \ldots, X_k) \leftarrow g_1, \ldots, g_n) & = & \mathtt{pushenv\ m} \\
 & & \mathrm{code}_G\ g_1\ \rho \\
 & & \ldots \\
 & & \mathrm{code}_G\ g_n\ \rho \\
 & & \mathtt{popenv}
\end{array}
$$

Clauses

Instruction pushenv m allocates stack space for local variables:




```
SP = FP + m;
```

Clauses

Example: Let

$$r \equiv \mathsf{a}(X, Y) \leftarrow \mathtt{f}(\bar{X}, X_1),\ \mathsf{a}(\bar{X}_1, \bar{Y})$$

Then $\text{code}_C\ r$ generates the code:

```
pushenv 3          call f/2          putref 2
mark A          A: mark B            call a/2
putref 1           putref 3       B: popenv
putvar 3
```

Predicates

- A predicate q/k is defined by a sequence of clauses $rr \equiv r_1 \dots r_f$.
- The translation of predicates is performed by the function code_P.
- If a predicate has just a single clause (ie. $f = 1$), we have:

$$\text{code}_P\ r \quad = \quad \text{code}_C\ r$$

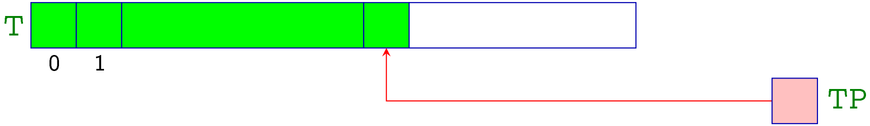
- If a predicate has several clauses, then:
  - we first "try" the first clause;
  - if it fails, then "try" the second one; etc.

Predicates

- If unification fails, we call the run-time function `backtrack()`.
- The goal is to roll back the whole computation to the *bactrack point*; ie. to the (dynamically) latest goal where there is another clause to "try".
- In order to restore previously valid bindings, we have used the run-time function `trail()` which stores new bindings in a special memory area.

Predicates

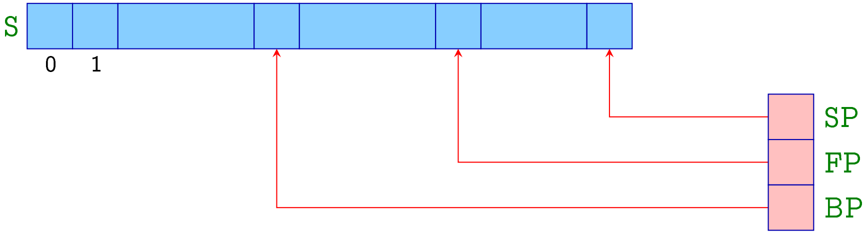
Trail:



T	=	Trail — memory area for storing new bindings;
TP	=	Tail-Pointer — points to the topmost used cell.

Predicates

There is also a special register BP which points to the current backtrack point.



Predicates

A bactrack point is a stack frame to which program execution possibly returns:

FP →		
	PosCont	0
	FPold	-1
	HPold	-2
	TPold	-3
	BPold	-4
	NegCont	-5

We will use following macros to denote organizational cells:

`PosCont`	$\equiv$	`S[FP]`	`TPold`	$\equiv$	`S[FP−3]`
`FPold`	$\equiv$	`S[FP−1]`	`BPold`	$\equiv$	`S[FP−4]`
`HPold`	$\equiv$	`S[FP−2]`	`NegCont`	$\equiv$	`S[FP−5]`

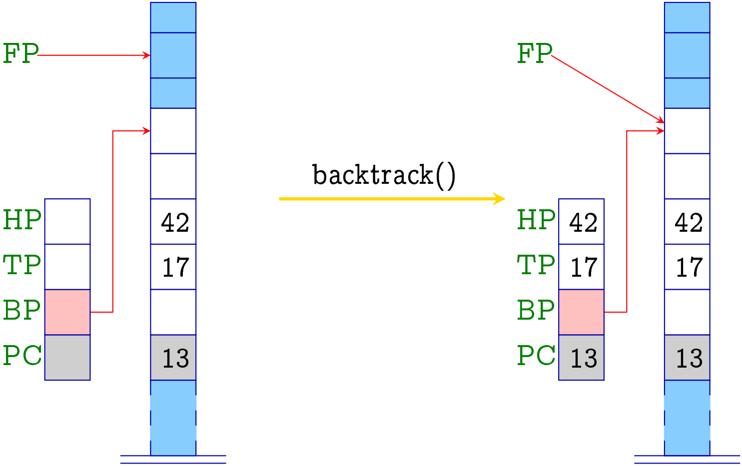
Predicates

The run-time function `backtrack()` restores registers according to the frame corresponding to backtrack point:

```
void backtrack() {
    FP = BP;
    HP = HPold;
    reset (TPold,TP);
    TP = TPold;
    PC = NegCont;
}
```

The function `reset()` restores variable bindingins; ie. undoes all bindings created after the backtrack point.

Predicates



Predicates

- The variables which are created since the last backtrack point can be removed together with their bindings simply by restoring the old value of the register HP.
- This works fine if *younger* variables always point to *older* objects.
- Bindings where *older* variables point to *younger* objects must be reset "manually".
- These bindings are recorded in the *trail*.

Predicates

The function `trail()` records a binding if the argument points to a younger object:

```
void trail (ref u) {
  if (u < S[BP-2]) {
    TP = TP+1;
    T[TP] = u;
  }
}
```

The cell `S[BP-2]` contains the value of HP before the creation of backtrack point.

The function `reset()` removes all bindings created after the last backtrack point:

```
void reset (ref x, ref y) {
  for (ref u=y; x<u; u--)
    H[T[u]] = (R,T[u]);
}
```

Predicates

Translation of a predicate q/k, which is defined by clauses $r_1, \ldots, r_f \quad (f > 1)$, generates a code which:

- creates a backtrack point;
- successively "tries" the alternatives;
- deletes the backtrack point.

Predicates

$$\texttt{code}_P\ (r_1, \ldots, r_f) \quad = \quad \begin{array}{lll} \texttt{q/k:} & \texttt{setbtp} & \qquad\qquad \texttt{jump A}_f \\ & \texttt{try A}_1 & \texttt{A}_1\texttt{:}\ \texttt{code}_C\ r_1 \\ & \ldots & \qquad\qquad \ldots \\ & \texttt{try A}_{f-1} & \texttt{A}_f\texttt{:}\ \texttt{code}_C\ r_f \\ & \texttt{delbtp} & \end{array}$$

NB!

- The backtrack point is deleted before the last alternative is "tried".
- For the "last try", the code jumps directly to the alternative and never returns to the present frame.

Predicates

Example:

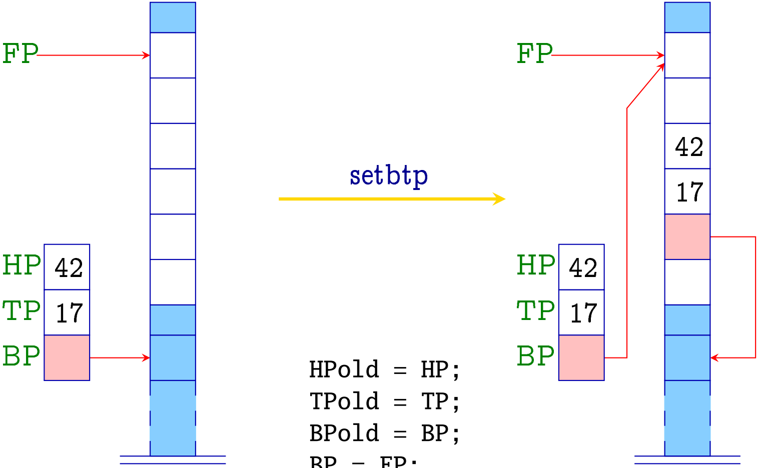
$$\begin{array}{lcl} \mathtt{s}(X) & \leftarrow & \mathtt{t}(\bar{X}) \\ \mathtt{s}(X) & \leftarrow & \bar{X} = a \end{array}$$

Translation of the predicate s/1 results:

```
s/1: setbtp        A: pushenv 1       B: pushenv 1
     try A            mark C             putref 1
     delbtp           putref 1           uatom a
     jump B           call t/1           popenv
                   C: popenv
```

Predicates

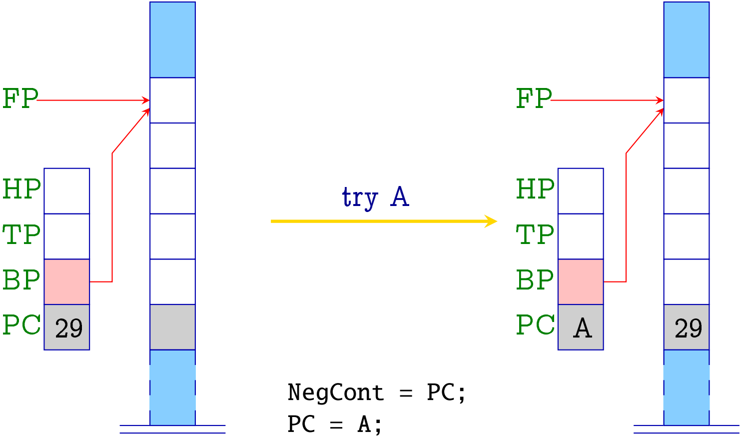
Instruction `setbtp` saves registers HP, TP, BP:




```
HPold = HP;
TPold = TP;
BPold = BP;
BP = FP;
```

Predicates

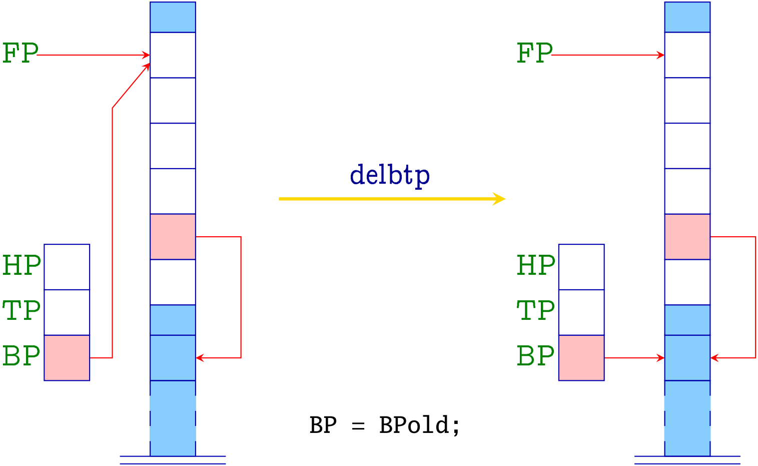
Instruction `try A` saves the current PC as the negative continuation address and jumps to the alternative to be "tried" at address A:




```
NegCont = PC;
PC = A;
```

Predicates

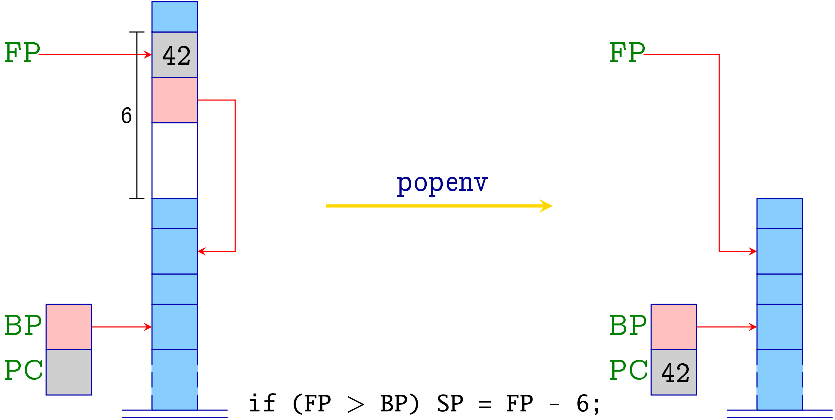
Instruction delbtp restores the value of BP:




```
BP = BPold;
```

Predicates

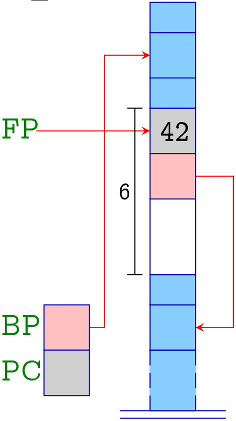
Instruction popenv restores registers FP and PC, and if possible pops the stack frame:

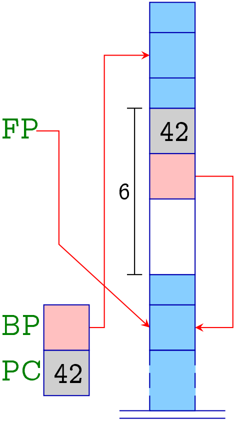



```
if (FP > BP) SP = FP - 6;
PC = PosCont;
FP = FPold;
```

Predicates

If FP $\leq$ BP the the frame is not deallocated:






```
if (FP > BP) SP = FP - 6;
PC = PosCont;
FP = FPold;
```

Queries and Programs

- Translation of a program $p \equiv rr_1 \ldots rr_h?g$ generates:
  - code for evaluating the query g;
  - code for the predicate definitions rr_i.
- Query evaluation is preceeded by:
  - initialization of registers;
  - allocation of space for globals.
- Query evaluation is succeeded by:
  - returning the values of globals.

Queries and Programs

$$\text{code}\ (rr_1 \ldots rr_h\ ?g) \quad = \quad \begin{array}{ll} \texttt{init} & \text{code}_P\ rr_1 \\ \texttt{pushenv d} & \ldots \\ \text{code}_G\ g\ \rho & \text{code}_P\ rr_h \\ \texttt{halt d} & \end{array}$$

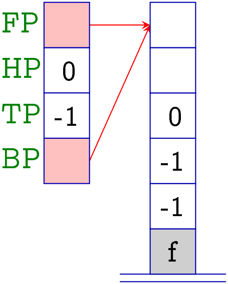
where $\mathit{free}(g) = \{X_1, \ldots, X_d\}$ and $\rho = \{X_i \mapsto i \mid i = 1 \ldots d\}$.

Instruction `halt d` ...

- ... terminates the program execution;
- ... returns the values of d globals;
- ... if user requests, performs backtracking.

Queries and Programs

Instruction `init` creates the initial backtrack point:




```
BP = FP = SP = 5;
S[0] = f;
S[1] = S[2] = -1;
S[3] = 0;
BP = FP;
```

If the query g fails, the code at address f will be executed (eg. prints a message telling about the failure).

Queries and Programs

Example:

$$\mathtt{t}(X) \leftarrow \bar{X} = b \qquad \mathtt{q}(X) \leftarrow \mathtt{s}(\bar{X}) \qquad \mathtt{s}(X) \leftarrow \bar{X} = a$$

$$\mathtt{p} \leftarrow \mathtt{q}(X),\ \mathtt{t}(\bar{X}) \qquad \mathtt{s}(X) \leftarrow \mathtt{t}(\bar{X}) \qquad ?\ \mathtt{p}$$

```
      init            p/0: pushenv 1     q/1: pushenv 1     E: pushenv 1
      pushenv 0            mark B             mark D            mark G
      mark A               putvar 1           putref 1          putref 1
      call p/0             call q/1           call s/1          call t/1
   A: halt 0            B: mark C          D: popenv         G: popenv
 t/1: pushenv 1            putref 1      s/1: setbtp         F: pushenv 1
      putref 1             call t/1           try E             putref 1
      uatom b           C: popenv             delbtp            uatom a
      popenv                                  jump F            popenv
```

Last Call Optimization

Consider the predicate app/3 defined as follows:

$$
\begin{array}{lcl}
\text{app}(X, Y, Z) & \leftarrow & X = [\,],\ Y = Z \\
\text{app}(X, Y, Z) & \leftarrow & X = [H \mid X'],\ Z = [H \mid Z'],\ \text{app}(X', Y, Z')
\end{array}
$$

The last goal of the second clause is a recurisive call:

- we can evaluate it in the current stack frame;
- after (successful) completion, we will not return to the current frame but go directly back to the "predecessor" frame.

Last Call Optimization

Consider a clause $r \equiv \mathrm{p}(X_1, \ldots, X_k) \leftarrow g_1, \ldots, g_n$, which has m local variables and where $g_n \equiv \mathrm{q}(t_1, \ldots, t_h)$.

$$
\begin{array}{lcll}
\mathtt{code}_C\ r & = & \mathtt{pushenv\ m} & \qquad \mathtt{code}_A\ t_1\ \rho \\
 & & \mathtt{code}_G\ g_1\ \rho & \qquad \ldots \\
 & & \ldots & \qquad \mathtt{code}_A\ t_h\ \rho \\
 & & \mathtt{code}_G\ g_{n-1}\ \rho & \qquad \mathtt{call\ q/h} \\
 & & \mathtt{mark\ B} & \mathrm{B:}\ \mathtt{popenv}
\end{array}
$$

Last Call Optimization

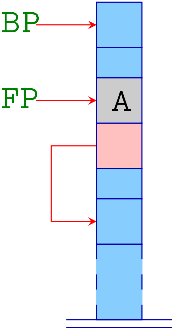
Consider a clause $r \equiv \mathrm{p}(X_1, \ldots, X_k) \leftarrow g_1, \ldots, g_n$, which has m local variables and where $g_n \equiv \mathrm{q}(t_1, \ldots, t_h)$.

$$
\begin{array}{lclll}
\texttt{code}_C\; r & = & \texttt{pushenv m} & & \texttt{code}_A\; t_1\; \rho \\
 & & \texttt{code}_G\; g_1\; \rho & & \ldots \\
 & & \ldots & & \texttt{code}_A\; t_h\; \rho \\
 & & \texttt{code}_G\; g_{n-1}\; \rho & & \texttt{lastcall (q/h,m)} \\
 & & \texttt{lastmark} & &
\end{array}
$$

Last Call Optimization

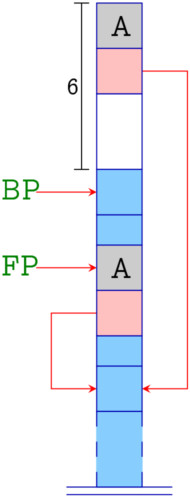
- If the current clause is not last or goals $g_1, \ldots, g_{n-1}$ have created backtrack points, then $\mathtt{FP} \leq \mathtt{BP}$.
- Then the instruction `lasmark` creates a new frame but stores a reference to the predecessor frame.
- Otherwise (ie. if $\mathtt{FP} > \mathtt{BP}$), it does nothing.

Last Call Optimization



lastmark $\longrightarrow$

```
if (FP ≤ BP) {
  SP = SP + 6;
  S[SP] = PosCont;
  S[SP-1] = FPold;
}
```

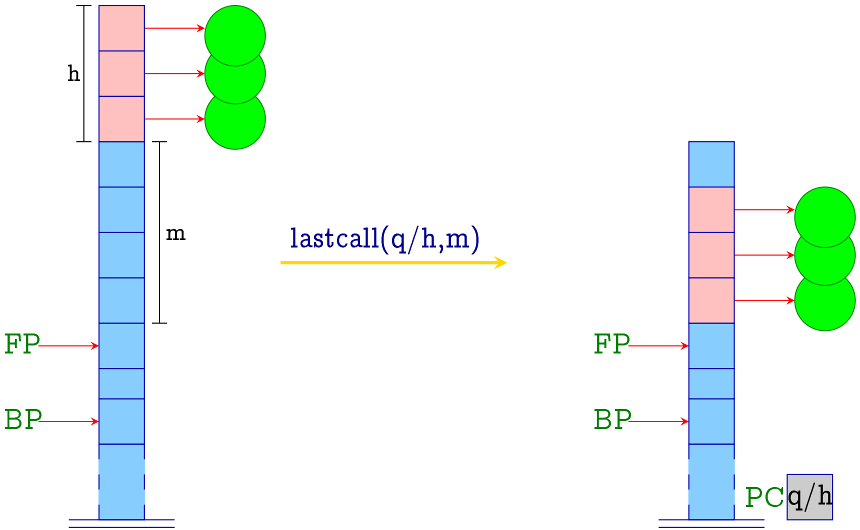


Last Call Optimization

- If FP $\leq$ BP, then the instruction `lastcall (q/h,m)` behaves like `call q/h`.
- Otherwise, the current stack frame is reused:
  - the cells S[FP+1], ..., S[FP+h] get new values;
  - and then directly jumps to the predicate q/h.

```
lastcall (q/h,m)  =  if (FP ≤ BP) call q/h;
                     else {
                         move (m,h);
                         jump q/h;
                     }
```

Last Call Optimization



Last Call Optimization

Consider the clause

$$\mathtt{a}(X, Y) \leftarrow \mathtt{f}(\bar{X}, X_1),\ \mathtt{a}(\bar{X}_1, \bar{Y})$$

The last call optmization yields:

```
   pushenv 3        putvar 3     putref 3
   mark A           call f/2     putref 2
   putref 1     A:  lastmark     lastcall(a/2,3)
```

NB! If the clause is last and its last goal is the only one, then we can omit `lastmark` and replace `lastcall(q/h,m)` with instructions `move(m,h)` and `jump q/h`.

Last Call Optimization

The last call optmization for the second clause of app/3 yields:

```
A:  pushenv 6
    putref 1
    ustruct [|]/2 B
    son 1
    uvar 4
    son 2
    uvar 5
    up C
B:  putvar 4
    putvar 5
    putstruct [|]/2
    bind
C:  putref 3
    ustruct [|]/2 D
    son 1
    uref 4
    son 2
    uvar 6
    up E
D:  check 4
    putref 4
    putvar 6
    putstruct [|]/2
    bind
E:  putref 5
    putref 2
    putref 6
    move(6,3)
    jump app/3
```

Stack Frame Trimming

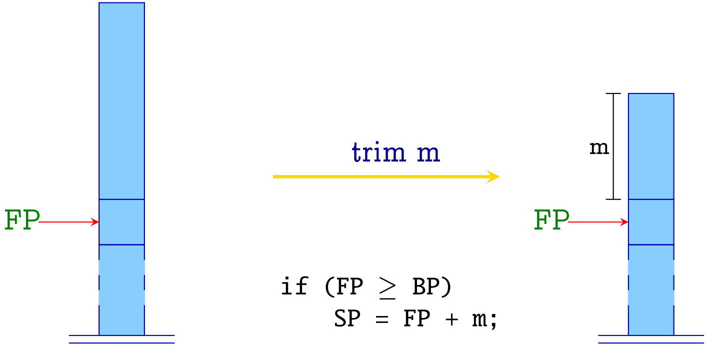
- Order local variables according to their *life time*.
- If possible, remove *dead* variables.
- Example:

$$\mathrm{a}(X, Z) \leftarrow \mathrm{p}_1(\bar{X}, X_1), \mathrm{p}_2(\bar{X}_1, X_2), \mathrm{p}_3(\bar{X}_2, X_3), \mathrm{p}_4(\bar{X}_3, Z)$$

  - after the goal $\mathrm{p}_2(\bar{X}_1, X_2)$ the variable X_1 is dead;
  - after the goal $\mathrm{p}_3(\bar{X}_2, X_3)$ the variable X_2 is dead.

Stack Frame Trimming

After every non-last goal which has dead variables insert an instruction `trim`:



NB! We can remove dead locals only if there are no new backtrack points created.

Stack Frame Trimming

Example:

$$a(X, Z) \leftarrow p_1(\bar{X}, X_1), p_2(\bar{X}_1, X_2), p_3(\bar{X}_2, X_3), p_4(\bar{X}_3, Z)$$

Ordering of the variables:

$$\rho = \{X \mapsto 1, Z \mapsto 2, X_3 \mapsto 3, X_2 \mapsto 4, X_1 \mapsto 5\}$$

```
    pushenv 5          putvar 4        C: trim 3
    mark A             call p2/2          lastmark
    putref 1        B: trim 4             putref 3
    putvar 5           mark C             putref 2
    call p1/2          putref 4           lastcall (p4/2,3)
 A: mark B             putvar 3
    putref 5           call p3/2
```

Clause Indexing

- Often, predicates are implemented by case distinction on the first argument.
- Hence, by inspecting the first argument, many alternatives can be excluded.
  - Failure is detected earlier.
  - Bactrack points are removed earlier.
  - Stack frames are removed earlier.

Clause Indexing

Example:

$$
\begin{array}{lcl}
\mathrm{app}(X, Y, Z) & \leftarrow & X = [\,],\ Y = Z \\
\mathrm{app}(X, Y, Z) & \leftarrow & X = [H \mid X'],\ Z = [H \mid Z'],\ \mathrm{app}(X', Y, Z')
\end{array}
$$

- If the first argument is `[]`, then only the first clause is applicable.
- If the first argument has `[|]` as its root constructor, then only the second clause is applicable.
- Every other root constructor of the first argument will fail.
- Both alternatives should be tried only if the first argument is uninitialized variable.

Clause Indexing

- Introduce a separate *try chain* for every possible constructor.
- Inspect the root node of the first argument.
- Depending on the result, perform an indexed jump to the appropriate try chain.

Let the predicate p/k defined by the sequence of clauses $rr \equiv r_1 \ldots r_m$.

The macro tchains rr denotes the sequence of try chains which correspond to the root constructors occuring in unifications $X_1 = t$.

Clause Indexing

Example:
Consider the predicate app/3. Let the code for its two clauses start at addresses A_1 and A_2. Then we get the following four try chains:

```
VAR:  setbtb  // variables       NIL:  jump A1  // []
      try A1                     CONS: jump A2  // [|]
      delbtp                     ELSE: fail     // default
      jump A2
```

Instruction fail handles all constructors besides [] and [|].

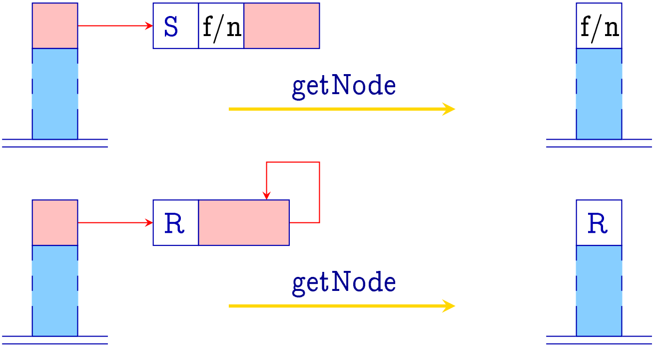
$$\texttt{fail} \quad = \quad \texttt{backtrack()}$$

Clause Indexing

Then we generate for a predicate p/k:

$$
\begin{array}{rcrl}
\mathrm{code}_P\ rr & = & & \mathtt{putref}\ 1 \\
 & & & \mathtt{getNode} \\
 & & & \mathtt{index}\ \mathtt{p/k} \\
 & & & \mathrm{tchains}\ rr \\
 & & \mathrm{A}_1: & \mathrm{code}_C\ r_1 \\
 & & & \ldots \\
 & & \mathrm{A}_m: & \mathrm{code}_C\ r_m
\end{array}
$$

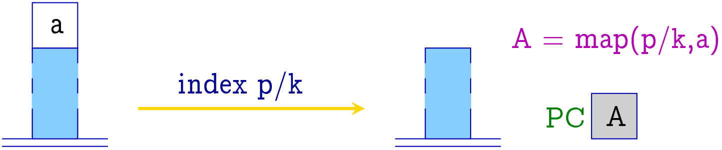
Clause Indexing




```
switch (H[S[SP]]) {
    case (S,f/n):   S[SP] = f/n; break;
    case (A,a):     S[SP] = a; break;
    case (R,_):     S[SP] = R;
}
```

Clause Indexing

Instruction index p/k performs an indexed jump to the appropriate try chain:




```
PC = map(p/k,S[SP]);
SP = SP - 1;
```

The function map() returns the start address of the appropriate try chain. Can be defined eg. through some hash table.

Cut Operator

We extend the language Proll with the cut operator "!" which explicitly allows to prune the search space of backtracking.

Example:

$$\begin{array}{lcl} \texttt{branch}(X,Y) & \leftarrow & \mathrm{p}(X),\ !,\ \mathrm{q}_1(X,Y) \\ \texttt{branch}(X,Y) & \leftarrow & \mathrm{q}_2(X,Y) \end{array}$$

If all the queries before the cut have succeeded, then the choice is *committed*: backtracking will return only to backtrack points *preceding* the call to the predicate.

Cut Operator

The cut operator should:

- restore the register BP by assigning to it BPold from the current frame;
- remove all frames which are on top of the local variables.

Accordingly, we translate the cut into the sequence:

```
prune
pushenv m
```

where m is the number of (still alive) local variables of the clause.

Cut Operator

Example:

$$\begin{array}{lcl} \texttt{branch}(X,Y) & \leftarrow & \texttt{p}(X),\ !,\ \texttt{q}_1(X,Y) \\ \texttt{branch}(X,Y) & \leftarrow & \texttt{q}_2(X,Y) \end{array}$$

We obtain:

```
    setbtp     A: pushenv 2     C: prune                 B: pushenv 2
    try A         mark C           pushenv 2                putref 1
    delbtp        putref 1         lastmark                 putref 2
    jump B        call p/1         putref 1                 move(2,2)
                                   putref 2                 jump q₂/2
                                   lastcall(q₁/2,2)
```

Cut Operator

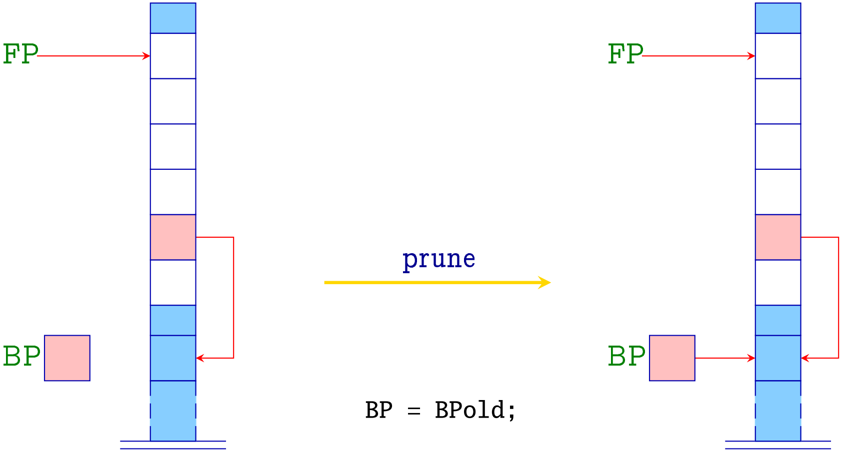
Example:

$$
\begin{array}{lcl}
\texttt{branch}(X,Y) & \leftarrow & \mathrm{p}(X),\ !,\ \mathrm{q}_1(X,Y) \\
\texttt{branch}(X,Y) & \leftarrow & \mathrm{q}_2(X,Y)
\end{array}
$$

... or, when using optimizations:

```
    setbtp      A: pushenv 2     C: prune           B: pushenv 2
    try A          mark C           pushenv 2          putref 1
    delbtp         putref 1         putref 1           putref 2
    jump B         call p/1         putref 2           move(2,2)
                                    move(2,2)          jump q2/2
                                    jump q1/2
```

Cut Operator



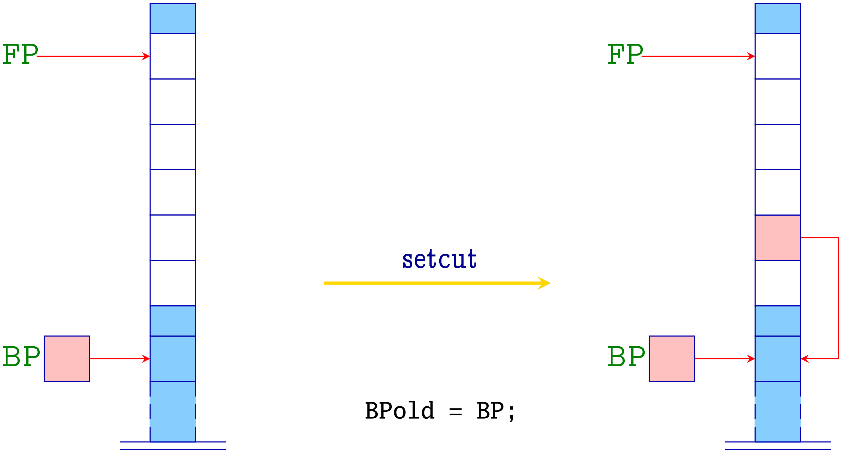
Cut Operator

Problem:

If the predicate is defined by a *single* clause, then we have not stored the old BP inside the stack frame.

For the cut to work also with single-clause predicates or try chains of length 1, we insert an extra instruction `setcut` before the clausal code (or the jump).

Cut Operator



Cut Operator

Final example: the predicate `notP` succeeds whenever `p` fails and vice versa:

$$
\begin{array}{lcl}
\texttt{notP}(X) & \leftarrow & \texttt{p}(X),\ !,\ \texttt{fail} \\
\texttt{notP}(X) & \leftarrow &
\end{array}
$$

where `fail` always fails.

```
setbtp     A: pushenv 1     C: prune          B: pushenv 1
try A         mark C           pushenv 1         popenv
delbtp        putref 1         fail
jump B        call p/1         popenv
```