

The war on error

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Syntax and basic definitions

Anatomy of a file

- A file usually starts with the first line
 - `module filename where`
- The rest of the file proceeds without indenting.
- Top level definitions must be declared before use.
- However, before the first line we can add options (usually to turn something off) which correspond to command line options:
 - `{-# OPTIONS --no-termination-check #-}`

Naming

- Agda is very liberal about naming:
- There are a limited number of reserved characters:
`@ . () { } ; _` which cannot be used at all in names
- and some reserved words which cannot be used in name parts:
 - `-> : = ? \ | →∀ λ abstract data forall hiding import in
infix infixl infixr let module mutual open postulate
primitive private public record renaming rewrite using
where with Prop Set[0-9]* [0-9]+`
- Names parts: Eg. `_part1_part2_` or `if_then_else_`

Spacing continued

- There are consequences of being liberal about naming.
- Notice that $:$ $=$ $?$ \backslash $|$ \rightarrow \forall λ are reserved words not characters so they can be part of names:
 - Eg. $x+z=x$, $S \rightarrow T$, $x : S$ are valid names!
- Instead we must use spacing between different clauses
 $x : S$ and $S \rightarrow T$ etc.
- Note that reserved characters do not need a space separator. Eg. $(x + y) + z$, `Module.function`

Comments

- single-line comments begin with `--`
- Eg. `f x = ? -- I'll fill this in later`
- Multi-line comments begin with `{-` and end with `-}`
- Eg. `{- The rest of this file is utterly incomprehensible -}`

Indenting

Indenting is used to structure the file into blocks like in Haskell, I recommend using two spaces each time. Eg.

```
data X : Set where
```

```
  con : X
```

and

```
  f : X → X
```

```
  f x = g x
```

```
  where
```

```
    g : X → X
```

```
    g x = x
```

Unlike Haskell this is the only option!

Unicode

- Unicode characters can be entered in TeX/LaTeX style. Eg. `\mu`, `\lambda`, `\Downarrow`
- Superscript `^` and subscript `_` are preceded by a `\`. Eg. `\pi_0`, `\sigma^1`
- To see how to type a character in emacs place the cursor on it and type `C-u C-x =`
- Warning! It's easy to get carried away with unicode but limited use of it is nice: greek symbols, and nice arrows.

Infix operators

- In Agda we can have mixfix operators Eg.
 - `_+_` : `Nat → Nat → Nat`
 - `if_then_else_` : `Bool → Bool → Bool → Bool`
- Type constructors, data constructors, functions can be named in this way.
- We can specify associativity and precedence for infix operators should be stated before use.
 - `infixl 5 _*_`
 - `infixr 6 _+_`
 - `infix 6 _==_`

Local definitions

$f : X \rightarrow X$

$f\ x = g\ x$

where

$g : X \rightarrow X$

$g\ x = x$

or

$f : X \rightarrow X$

$f\ x = g\ x$

where

$g : X \rightarrow X$

$g\ x = x$

and

$h : X \rightarrow X$

$h\ x = \text{let } g : X \rightarrow X$

$g\ x = x$

in $g\ x$

$g\ x = x$

Built-in types

You can declare Char, String, and Nat as built-in. Then you can use nicer syntax

```
postulate Char : Set
{-# BUILTIN CHAR Char #-}
```

```
postulate String : Set
{-# BUILTIN STRING String #-}
```

```
data Nat ...
{-# BUILTIN NATURAL Nat #-}
{-# BUILTIN ZERO z #-}
{-# BUILTIN SUC s #-}
```

λ -expression

The polymorphic identity function

$id : \{X : Set\} \rightarrow X \rightarrow X$

$id\ x = x$

can be rewritten as (the scope is extends as far as possible):

$id = \lambda\ x \rightarrow x$

but we cannot pattern match on the RHS and there is not case expression like in Haskell

~~$f = \lambda\ (con\ x\ y) \Rightarrow x$~~

~~$f\ x = case\ x\ of\ (con\ y\ z) \Rightarrow x$~~

Note also that let is a genuine expression and can go under a lambda but where is not and can't.

Implicit arguments

Implicit arguments are stated in types with $\{ \}$ instead of $()$

```
id : { X : Set } → X → X
```

```
id x = x
```

the type can sometimes be omitted and inferred by Agda

```
id : ∀ { X } → X → X
```

and the argument can be made explicit in the LHS

```
id { X } x = x
```

on the RHS any explicit argument can be left implicit by putting an `_` but Agda might not know the answer!

```
id x = _
```

Inductive data types

```
data tcon1 : Set where  
  dcon1 : tcon1  
  dcon2 : tcon1 → tcon1
```

```
data tcon2 : Set where  
  dcon3 : tcon2  
  dcon4 : tcon2
```

```
data tcon3 : Set where  
  dcon5 : tcon3
```

```
data tcon4 : Set where
```

What types are these?

Parameters and indices

```
data Vec (A : Set) : Nat → Set where
  []      : Vec A z
  _ :: _  : ∀ {n} → A → Vec A n → Vec A (s n)
```

- Parameters come before the colon in the data declaration and are in scope of the constructors and don't vary.
- Indices come after and are not automatically in scope and can vary.

Postulates

- In Agda we can assume something without proof using a postulate
- We can delay a proof by just putting a ? but a postulate is taken as an axiom that we do not intend to prove later.
- Maybe we can't prove it at all: Eg.

- `postulate EM : $\forall (X : Set) \rightarrow X \vee \neg X$`

Records

We can define pairs in two ways in Agda. The first is the one you already know using a data type

```
data _ × _ (A B : Set) : Set where
  _ , _ : A → B → A × B
```

we could then define projections as follows

```
fst : {A B : Set} → A × B → A
fst (a , _) = a
```

```
snd : {A B : Set} → A × B → B
snd (_, b) = b
```

Records

The second is to use a record:

```
record  $\_ \times \_$  (A B : Set) : Set where
  constructor  $\_ , \_$ 
  field      fst : A
            snd : B
```

open $_ \times _$ -- brings fst, snd into scope

We get projections automatically with the same name as the fields

$$\text{fst} : \{A B : \text{Set}\} \rightarrow A \times B \rightarrow A$$

The downside is we can't pattern match on records.

Σ -type (dependent pair)

- I have already told you about the Π -type which is the dependent function type.
- There is a dependent version of the pair type - the Σ -type. This can be defined as a record as follows:

```
record  $\Sigma$  (A : Set) (B : A  $\rightarrow$  Set) where
  constructor
  field
    fst : A
    snd : B fst
```

It is the logical counterpart the “there exists” \exists

Σ -type (dependent pair)

Or as a data type

```
data  $\Sigma$  (A : Set) (B : A  $\rightarrow$  Set) : Set where  
  _,_ : (a : A)  $\rightarrow$  B a  $\rightarrow$   $\Sigma$  A B
```

with projections

```
fst :  $\forall$  {A B}  $\rightarrow$   $\Sigma$  A B  $\rightarrow$  A
```

```
fst (a , _) = a
```

```
snd :  $\forall$  {A B} (p :  $\Sigma$  A B)  $\rightarrow$  B (fst p)
```

```
snd (_, b) = b
```