Covering the Path Space: A Casebase Analysis for Mobile Robot Path Planning

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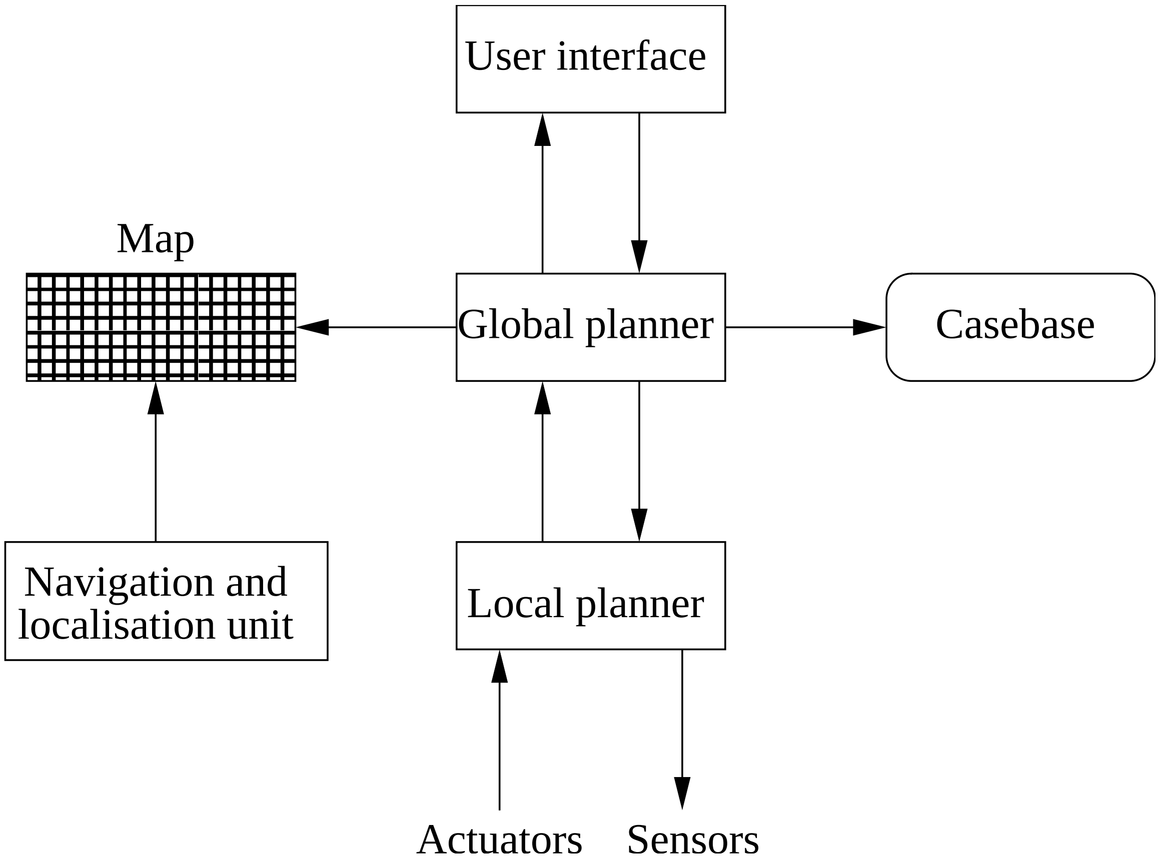
Path planning

In many real-life applications it is necessary for an autonomous agent to find a path between two points. The environment can be

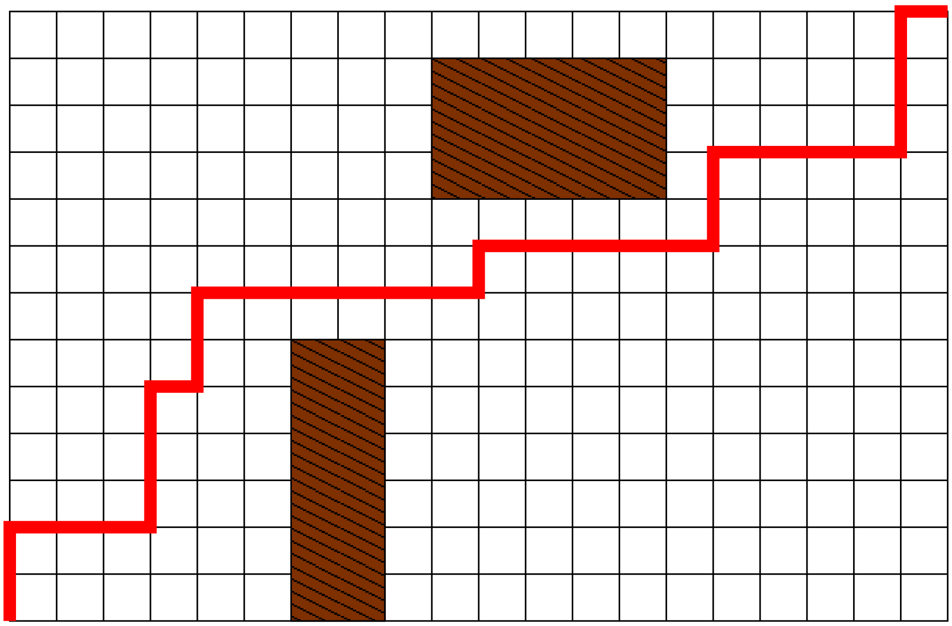
- complicated;
- unknown beforehand; or even
- dynamically changing.

System description



The world model

. . . is a grid map



Case-based reasoning

- Case is a segment-composed path from the lower-left corner of the world to the upper-right one together with its evaluation.
  - Speed of traversal, deviation, . . .

- Learning is performed through accumulating new cases.

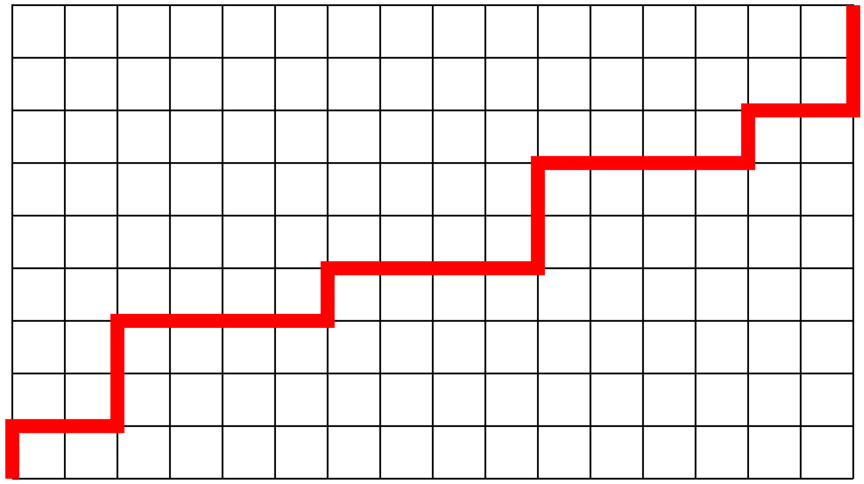
- For the sake of efficiency we would like to avoid the occurrance of too “similar” cases in the casebase.

Estimating the size of the casebase

- If we preplan the casebase so that for every potential path there is a “close” path in casebase, then how small can the casebase be?

- If we generate new cases on the fly and check that new cases are not too “close” to the old ones, then how large can the casebase be?

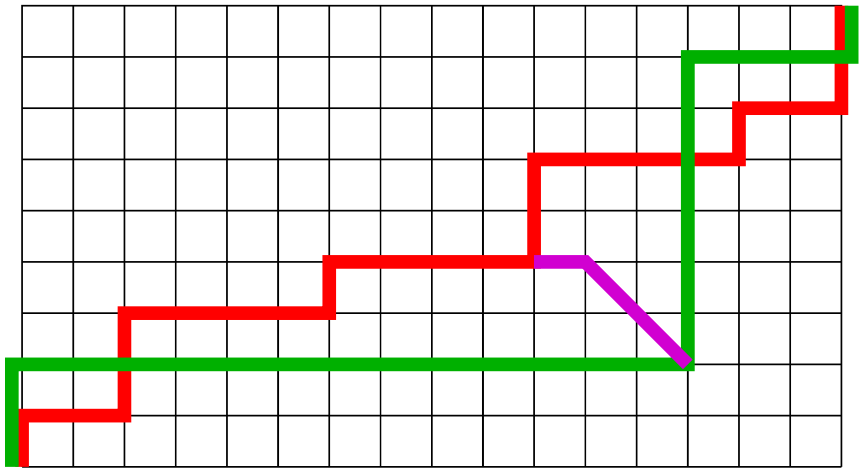
Grid paths



The set of such paths will be denoted by $\mathcal{P}_{m,n}$. It can be proven that

$$\pi(m,n) = |\mathcal{P}_{m,n}| = \binom{m+n}{m}.$$

What does “close” mean?



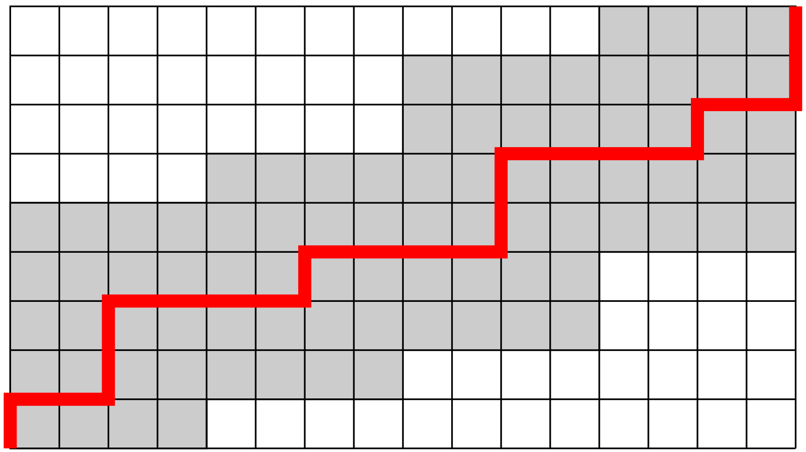
$$d_g(P_1, P_2) = \max_{c_1 \in P_1} \{ \min_{c_2 \in P_2} \{ d(c_1, c_2) \} \},$$

where $d(c_1, c_2) = \max\{|x_1 - x_2|, |y_1 - y_2|\}$ denotes the $\mathbb{R}^2_\infty$-distance for $c_1 = (x_1, y_1)$, $c_2 = (x_2, y_2)$.

Metric space $(\mathcal{P}_{m,n}, d_g)$

$(\mathcal{P}_{m,n}, d_g)$ is a metric space (this is *not* the case for Euclidean metrics). We have balls of paths in this space:

$$B(P, \delta) = \{P' \in \mathcal{P}_{m,n} : \; d_g(P, P') \leq \delta\}.$$



Main problem statement

What are lower and upper estimates for the cardinality of set S such that

$$\bigcup_{P \in S} B(P, \delta) = \mathcal{P}_{m,n}, \tag{1}$$

$$\forall P' \in S \left[P' \notin \bigcup_{P \in S \setminus \{P'\}} B(P, \delta) \right]. \tag{2}$$

Theorem 1

For every $\delta \in \mathbb{N}$ and every subset $S \subseteq \mathcal{P}_{m,n}$ satisfying the properties (1) and (2), the inequality

$$|S| \geq \pi\left(\left\lfloor \frac{m}{2\delta+1} \right\rfloor, \left\lfloor \frac{n}{2\delta+1} \right\rfloor\right)$$

holds. Evenmore, there exists such a set S that the properties (1) and (2) are satisfied and equality holds in the above inequality.

Theorem 2

For every $\delta \in \mathbb{N}$ and every subset $S \subseteq \mathcal{P}_{m,n}$ satisfying the properties (1) and (2), the inequality

$$|S| \leq \begin{cases} \pi\left(\left\lfloor \frac{m}{\delta} \right\rfloor, \left\lfloor \frac{n}{\delta} \right\rfloor\right), & \text{if } \delta \text{ is odd} \\ \\ \pi\left(\left\lfloor \frac{m}{\delta+1} \right\rfloor, \left\lfloor \frac{n}{\delta+1} \right\rfloor\right), & \text{if } \delta \text{ is even} \end{cases}$$

holds. Evenmore, there exists such a set S that the properties (1), (2) and $|S| = \pi\left(\left\lfloor \frac{m}{\delta+1} \right\rfloor, \left\lfloor \frac{n}{\delta+1} \right\rfloor\right)$ are satisfied.

Some computations

In our experiments we used parameters $m = 51$, $n = 67$ and $\delta = 5$. For the lower estimate we have

$$\pi\left(\left\lfloor\frac{51}{2\cdot 5+1}\right\rfloor, \left\lfloor\frac{67}{2\cdot 5+1}\right\rfloor\right) = \binom{4+6}{4} = 210.$$

The upper estimate is

$$\pi\left(\left\lfloor\frac{51}{5}\right\rfloor, \left\lfloor\frac{67}{5}\right\rfloor\right) = \binom{10+13}{10} = 1144066.$$

Conclusions

- If δ is reasonably chosen, it is possible to seed the casebase with paths so that the whole path space is covered.

- Even for a reasonably chosen δ it may happen that without managing the casebase, the number of paths in the casebase becomes too large to handle.