Algorithmic Generation of Path Fragment Covers for Mobile Robot Path Planning

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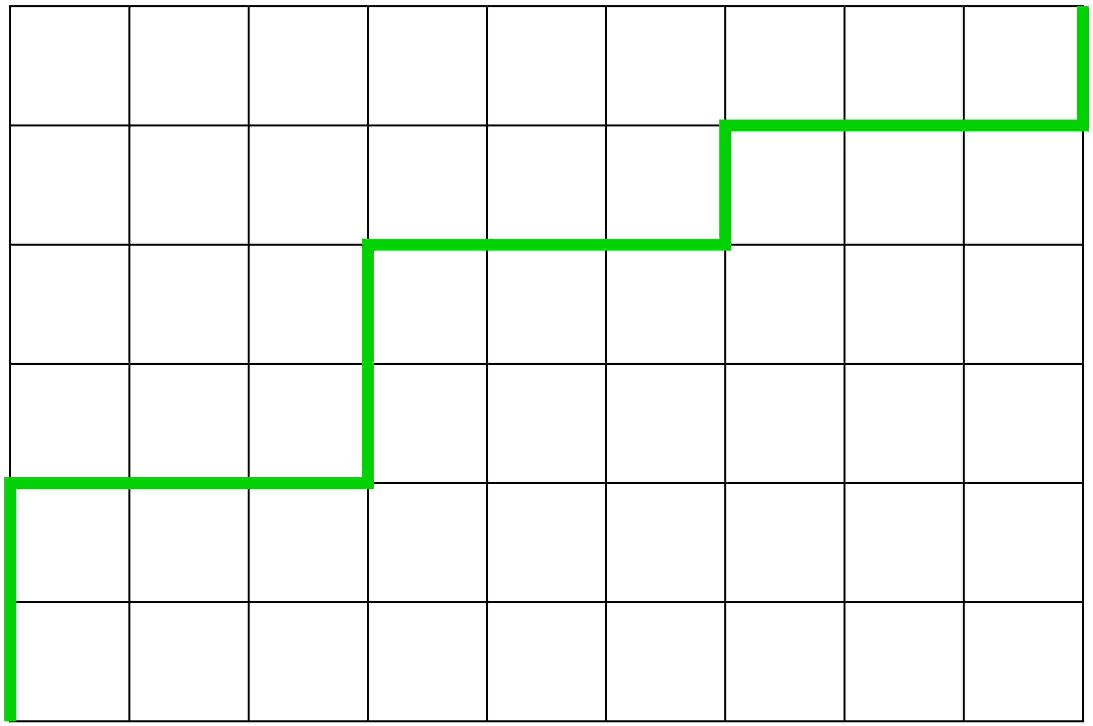
Motivation

- We consider mobile robot path planning in (semi-)dynamic environments
- The task of the robot is to traverse repetedly between predefined points, covering some given area, e.g. for
  - surveillance
  - cleaning tasks
  - demining applications
- At the same time, the robot must complete its task safely, minimizing the number of replannings and collisions with obstacles

The Model

- We model the robot's environment as an $m \times n$ grid map, where the robot moves from lower left to upper right corner moving only right and up along the grid lines



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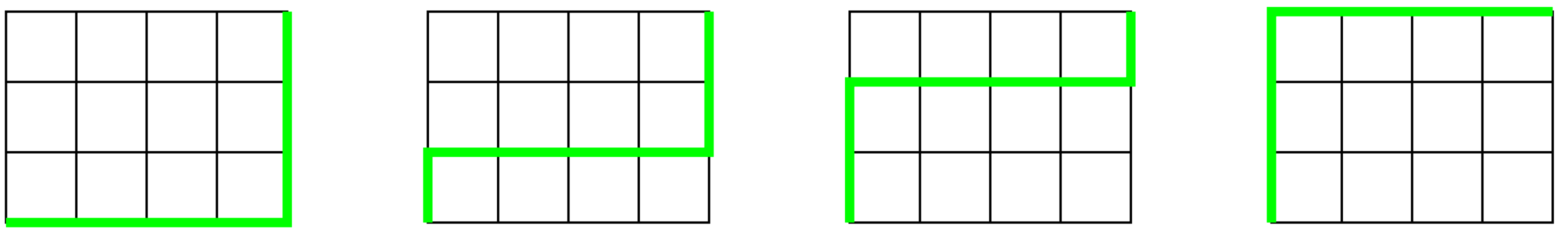

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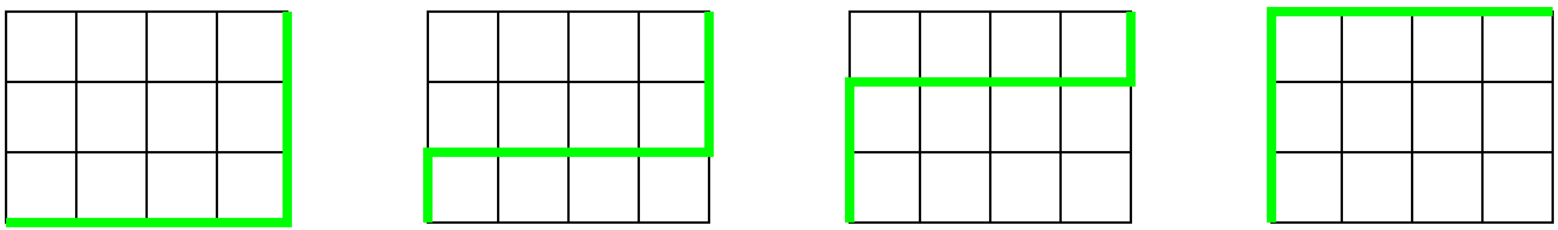
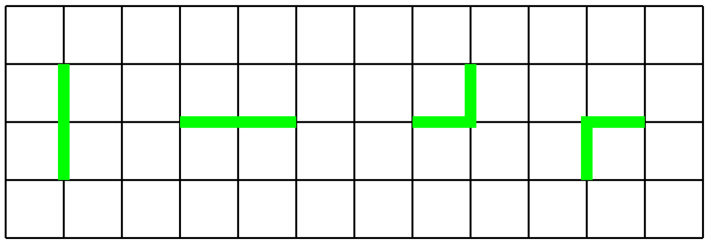


- It is possible to prove that the approach of defining distance between grid paths and trying to cover balls of paths does not scale well

Covering Path Fragments

- We define the path fragments (of length 2) to be the following unions of grid fragments:



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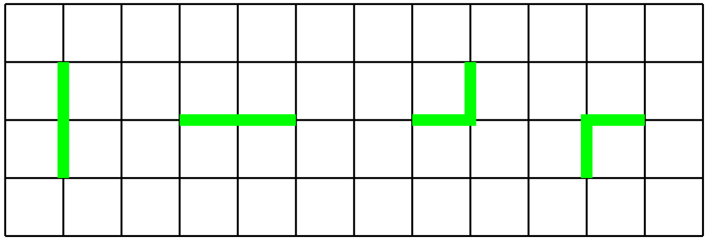

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- A subset $X \subseteq \mathcal{P}_{m,n}$ is called a *path fragment cover* if for every possible path fragment there is a path in X containing it

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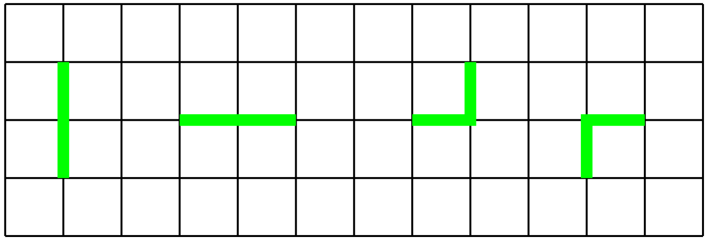

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- A subset $X \subseteq \mathcal{P}_{m,n}$ is called a *path fragment cover* if for every possible path fragment there is a path in X containing it
- **The main objective of the paper:** *Find the minimal size of a path fragment cover for the* $m \times n$ *grid. Describe all the path fragment covers of minimal size and design the means for generating them*

Minimal Path Fragment Cover

- **Theorem 1.** Cardinality $\tau(m,n)$ of the minimal path fragment cover of the the $m \times n$ grid is

$$\tau(m,n) = \begin{cases} 1, & \text{if } m = 0 \vee n = 0; \\ m+n, & \text{if } (m = 1 \,\&\, n \geq 1) \vee \\ & (n = 1 \,\&\, m \geq 1); \\ 2m+2n-2, & \text{if } m,n \geq 2 \end{cases}$$

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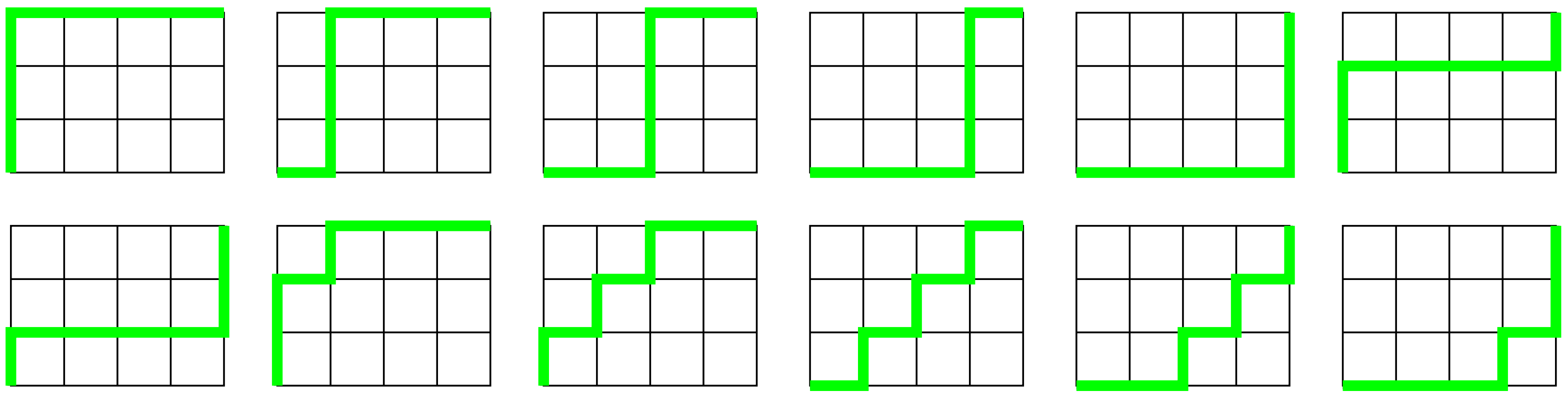
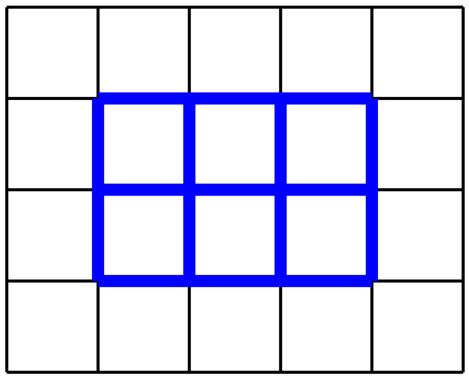
- An example of the *trivial* cover:



Describing the Minimal Covers

- It turns out that every minimal cover can be described by its behaviour on the set $\mathcal{S}_{m,n}$ of grid segments that have no endpoint on the grid's boundary:



- Clearly, $|\mathcal{S}_{m,n}| = (m-1)(n-2) + (m-2)(n-1)$

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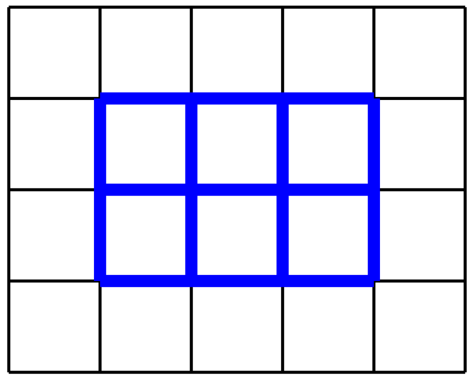
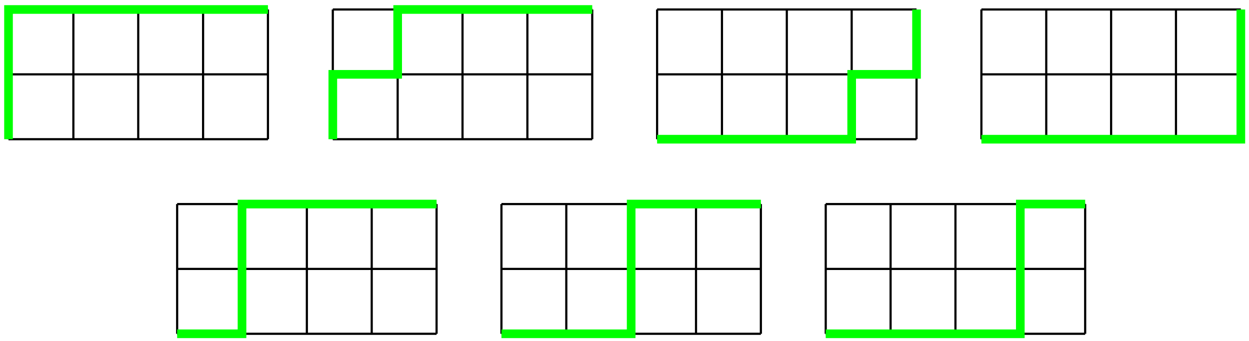


- Clearly, $|\mathcal{S}_{m,n}| = (m-1)(n-2) + (m-2)(n-1)$
- We will establish a one-to-one correspondence between minimal path fragment covers and functions from the set $F_{m,n} = \{f : \mathcal{S}_{m,n} \to \{0,1\}\}$

The Algorithm: Initialization

- First we find the paths common to all minimal covers – these are the ones that do not have an edge in $\mathcal{S}_{m,n}$
- For $m = 4$, $n = 2$ we get the following 7 paths:



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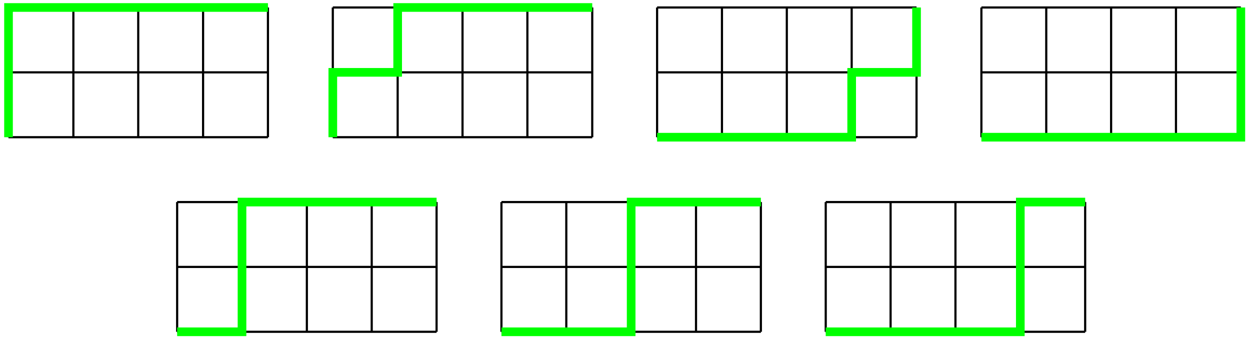
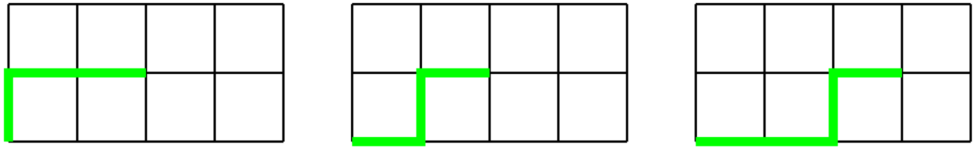


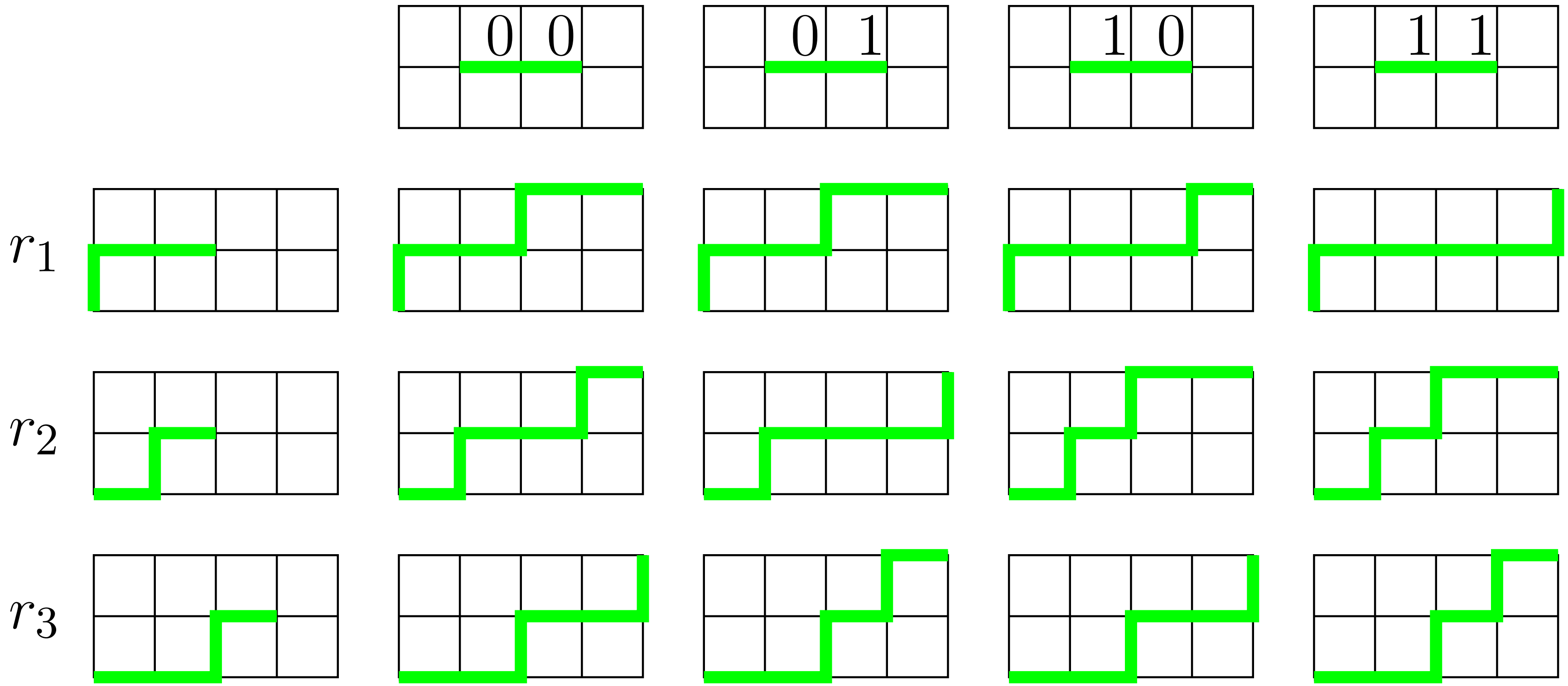
- Next we choose initial parts $r_1, r_2, \ldots, r_k$ of the other paths up to the first segment in $\mathcal{S}_{m,n}$
- For $m = 4$, $n = 2$ we get the following 3 parts:



The Algorithm: Main Cycle

- INPUT: Function $f \in F_{m,n}$ and path parts $r_1, r_2, \ldots, r_k$
- FOR each path part r_i ($i = 1, 2, \ldots, k$) DO
  - WHILE the last segment e of the current path part belongs to $\mathcal{S}_{m,n}$ DO
    - IF $f(e) = 0$ and e has not been traversed before OR $f(e) = 1$ and e has been traversed before
      - THEN the next segment is upwards
      - ELSE the next segment is rightwards
  - Completing the WHILE cycle we have reached the boundary of the grid; complete the path to the upper right corner and OUTPUT it

The Algorithm: Illustration



The Algorithm: Propositions

- The Algorithm produces a minimal path fragment cover and conversely, all the minimal path fragment covers are produced this way
- There are

$$|F_{m,n}| = 2^{|\mathcal{S}_{m,n}|} = 2^{(m-1)(n-2)+(m-2)(n-1)}.$$

minimal path fragment covers of an $m \times n$ grid.

- Time complexity of the Algorithm is $O((m+n)^2)$ and its space complexity is $O(mn)$
    - Note that the same time complexity is needed for the result printout alone, thus the Algorithm is very efficient

Conclusions and Further Work

- We defined a notion for grid covering that provides linear cardinality covers and is thus very scalable
- We presented an efficient algorithm for minimal path fragment generation
- As a next step, measures for minimal cover evaluation must be defined, the best theoretical covers must be found and experimentally tested

Thank You!

Questions?