Algorithmic Generation of Path Fragment Covers for Mobile Robot Path Planning

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Motivation

- We consider mobile robot path planning in (semi-)dynamic environments.
- The task of the robot is to traverse repeatedly between predefined points, covering some given area, e.g. for surveillance, cleaning tasks, and demining applications.
- At the same time, the robot must complete its tasks safely, minimizing the number of replannings and collisions with obstacles.
We model the robot's environment as an $m \times n$ grid map, where the robot moves from lower left to upper right corner moving only right and up along the grid lines.
The Problem

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It is possible to prove that the approach of defining distance between grid paths and trying to cover balls of paths does not scale well.
Covering Path Fragments

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- Let $P_{m,n}$ denote the set of all legal grid paths

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- We define the path fragments (of length 2) to be the following unions of grid fragments:

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- A subset $X \subseteq P_{m,n}$ is called a *path fragment cover* if for every possible path fragment there is a path in $X$ containing it

- **The main objective of the paper**: Find the minimal size of a path fragment cover for the $m \times n$ grid. Describe all the path fragment covers of minimal size and design the means for generating them
Theorem 1. Cardinality $\tau(m, n)$ of the minimal path fragment cover of the $m \times n$ grid is

$$
\tau(m, n) = \begin{cases} 
1, & \text{if } m = 0 \lor n = 0; \\
 m + n, & \text{if } (m = 1 \& n \geq 1) \lor (n = 1 \& m \geq 1); \\
2m + 2n - 2, & \text{if } m, n \geq 2 
\end{cases}
$$

Minimal Path Fragment Cover

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\end{cases}
$$

An example of the *trivial* cover:
Describing the Minimal Covers

It turns out that every minimal cover can be described by its behaviour on the set $S_{m,n}$ of grid segments that have no endpoint on the grid’s boundary:

Clearly, $|S_{m,n}| = (m - 1)(n - 2) + (m - 2)(n - 1)$
Describing the Minimal Covers

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\]

Clearly, \( |S_{m,n}| = (m - 1)(n - 2) + (m - 2)(n - 1) \)

We will establish a one-to-one correspondence between minimal path fragment covers and functions from the set \( F_{m,n} = \{f : S_{m,n} \rightarrow \{0, 1\}\} \)
The Algorithm: Initialization

First we find the paths common to all minimal covers – these are the ones that do not have an edge in $S_{m,n}$

For $m = 4$, $n = 2$ we get the following 7 paths:
The Algorithm: Initialization

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- For $m = 4$, $n = 2$ we get the following 7 paths:
  ![Paths](image)

- Next we choose initial parts $r_1, r_2, \ldots, r_k$ of the other paths up to the first segment in $S_{m,n}$
- For $m = 4$, $n = 2$ we get the following 3 parts:
  ![Initial Parts](image)
The Algorithm: Main Cycle

- INPUT: Function \( f \in F_{m,n} \) and path parts \( r_1, r_2, \ldots, r_k \)
- FOR each path part \( r_i \ (i = 1, 2, \ldots, k) \) DO
  - WHILE the last segment \( e \) of the current path part belongs to \( S_{m,n} \) DO
    - IF \( f(e) = 0 \) and \( e \) has not been traversed before OR \( f(e) = 1 \) and \( e \) has been traversed before
      - THEN the next segment is upwards
      - ELSE the next segment is rightwards
  - Completing the WHILE cycle we have reached the boundary of the grid; complete the path to the upper right corner and OUTPUT it
The Algorithm: Illustration

\[ \begin{array}{cccc}
0 & 0 & 0 & 1 \\
0 & 1 & 1 & 0 \\
1 & 0 & 1 & 1 \\
1 & 1 & 0 & 0 \\
\end{array} \]

\( r_1 \)

\( r_2 \)

\( r_3 \)
The Algorithm: Propositions

- The Algorithm produces a minimal path fragment cover and conversely, all the minimal path fragment covers are produced this way.

- There are

\[ |F_{m,n}| = 2|S_{m,n}| = 2(m-1)(n-2) + (m-2)(n-1). \]

minimal path fragment covers of an \( m \times n \) grid.

- Time complexity of the Algorithm is \( O((m + n)^2) \) and its space complexity is \( O(mn) \).

- Note that the same time complexity is needed for the result printout alone, thus the Algorithm is very efficient.
Conclusions and Further Work

- We defined a notion for grid covering that provides linear cardinality covers and is thus very scalable.
- We presented an efficient algorithm for minimal path fragment generation.
- As a next step, measures for minimal cover evaluation must be defined, the best theoretical covers must be found and experimentally tested.
Thank You!

Questions?