Counting and enumerating transformation monoids

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Motivated by the question of a colleague about the number of transformation monoids on a three-element set, we computed the answer 699. With the help of this number we were able to find the quite recent sequence `A343140` in Sloane's *Online Encyclopedia of Integer Sequences* (OEIS): '$1, 6, 699$' – no further entries are listed there. We show how to use Ganter's NEXTCLOSURE algorithm to extend this sequence by another term, that is, we enumerate and thus compute the number of all transformation monoids on a four-element set. We also use our enumeration approach to obtain related sequences for carrier sets of cardinality at most four: the number of all transformation monoids containing all constant operations (*constantive*), as well as of those that do not (*non-constantive*), and of those that avoid constant operations entirely (*constant-free*).