Right coherency for monoids

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A monoid S may be represented via mappings of sets or, equivalently and more concretely, by *S-acts*. A right S-act is a set A together with a map $A \times S \to A$ where $(a, s) \mapsto as$, such that for all $a \in A$ and $s, t \in S$ we have $a1 = a$ and $(as)t = a(st)$. We say that S is *right coherent* if every finitely generated S-subact of every finitely presented right S-act is finitely presented. This notion may equally be phrased in terms of *right congruences* on S.

Right coherency is a fascinating and elusive property. It arises naturally from several directions, including model theory and ring theory. Indeed the notion of right coherency for a monoid is analogous to that for a ring R (where, of course, S-acts are replaced by R-modules). Unlike the case for rings, it cannot be determined by properties of right ideals - and we must develop new strategies.

This talk will examine the notion of coherency from several angles: how it arises, its connection with other finitary properties and its interplay with so-called purity properties of S-acts. By the latter we mean conditions relating to solutions of sets of equations or, equivalently, to properties related to injectivity. It is natural to ask which monoids are right coherent - we present a new method which enables us to settle that question for some natural monoids of transformations.

[1]This work is drawn from a number of sources, the most recent being joint with Yang Dandan, and with Matthew Brookes and Nik Ruškuc.