Use of partial automorphisms in determining symmetry levels of combinatorial structures

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Even though researchers often tend to focus on combinatorial structures possessing many symmetries, the majority of combinatorial structures (for instance graphs) are in fact asymmetric, i.e., having no non-trivial symmetries at all. In our talk, we attempt to reconcile these two seemingly opposing views, and we will argue that asymmetric and highly symmetric structures are not that far apart as it may seem. For example, removing just a single vertex from a vertex transitive graph may result in a graph with a trivial automorphism group; while removing a vertex from a graph belonging to the family of minimal asymmetric graphs (introduced by Nešetřil) always leads to a graph with a non-trivial automorphism group. Furthermore: every proper induced subgraph on at least two vertices of a graph from this family has non-trivial automorphisms. Such situations call for the use of the concept of a partial automorphism which is an isomorphism between two induced substructures.

The set of all partial automorphisms together with the operation of partial composition forms an inverse monoid which is an analouge of the concept of an automorphism group that, we believe, better captures the local properties of the considered combinatorial structures. We will present the framework of inverse monoids of partial automorphisms and use the ratio between the maximal rank of a nontrivial partial automorphism and the order of the structure as a measure of its asymmetricity. In the end, we will also address some related algorithmic problems.