

On the semigroup of 2×2 matrices over a linearly ordered abelian group

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Our research is motivated by the article [1] by Johnson and Kambites. In that article, so-called tropical matrices are considered. These are matrices over $\mathbb{R} \cup \{-\infty\}$.

We consider 2×2 matrices over a linearly ordered abelian group \mathbf{A} . The set of all such matrices forms a semigroup with respect to multiplication

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} (a+e) \vee (b+g) & (a+f) \vee (b+h) \\ (c+e) \vee (d+g) & (c+f) \vee (d+h) \end{pmatrix}.$$

We discuss the properties of this semigroup. In particular, we concentrate on the Green's relations $\mathcal{R}, \mathcal{L}, \mathcal{H}, \mathcal{D}$ and \mathcal{J} in this semigroup. Among other things we give a necessary and sufficient condition for two matrices to be in \mathcal{D} -relation and we prove that $\mathcal{D} = \mathcal{J}$.

References

- [1] M. Johnson, M. Kambites, Multiplicative structure of 2×2 tropical matrices, *Linear Algebra Appl.* 435, 2011, 1612–1625.

¹Joint work with Marilyn Kutti