Near-unanimity closed minions of Boolean functions

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Let C_1 and C_2 be clones on sets A and B, respectively. A set K of functions of several arguments from A to B is called a (C_1, C_2)-*clonoid* if $KC_1 \subseteq K$ and $C_2K \subseteq K$. Sparks [2] classified the clones C on $\{0, 1\}$ according to the cardinality of the lattice $\mathcal{L}_{(\mathsf{J}_A, C)}$ of (J_A, C)-clonoids (here J_A denotes the clone of projections on a finite set A).

By Sparks's result, $\mathcal{L}_{(\mathsf{J}_A, C)}$ is finite if and only if C contains a near-unanimity operation. We sharpen this result by completely describing the lattices $\mathcal{L}_{(\mathsf{J}_A, C)}$ and the (J_A, C)-clonoids when $A = \{0, 1\}$ and C contains a near-unanimity operation. The case when C contains a majority operation was treated in [1]. In this talk, we will focus on the remaining clones containing a near-unanimity operation of arity greater than 3 but no majority operation. The key notion to the description is that of k-locally closed minorant minion, a set of functions closed under minors and minorants.

References

[1] E. Lehtonen, Majority-closed minions of Boolean functions, arXiv: 2102.01959.

[2] A. Sparks, On the number of clonoids, *Algebra Universalis* 80, 2019, no. 4, Paper No. 53, 10 pp.