On Centralizing Monoids with Majority Operation Witnesses

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Operations are considered over a fixed finite set A with $|A| > 2$. A *centralizing monoid* M is a set of unary operations which commute with some set F of operations. F is called a *witness* of M. A remarkable fact on a 3-element set $A\,(= E_3 = \{0, 1, 2\})$ is that a centralizing monoid is maximal (in the set of centralizing monoids) if and only if it has a constant operation or a majority minimal operation as its witness ([1]).

With respect to conjugacy, majority minimal operations on E_3 are divided into three classes. Denoting by W the set of triples in E_3^3 whose components are mutually distinct, representatives of three classes may be chosen, and called m_i $(i = 1, 2, 3)$, as follows: m_1 is constant on W taking the value 0, m_2 is 2-valued on W whose value is 0 on $\{(0, 1, 2), (1, 2, 0), (2, 0, 1)\}$ and 1 otherwise and m_3 behaves like the projection e_1^3 on W.

In this talk we examine the possibility of generalizing the above property from 3-element set to k-element set for any finite $k \geq 3$. Concerning the maximality, we obtain the following: For m_1, the centralizing monoid having the generalization of m_1 as its witness is always maximal. For m_3, the centralizing monoid with the generalized witness is maximal with one exception of $k = 4$. For m_2, a generalization itself is less obvious. We choose one majority operation, called m_{b}, which generalizes m_2 in a natural way, and explicitly describe the centralizing monoid having m_{b} as its witness. It turns out that it is *not* maximal for each $k \geq 4$, contrary to the case of $k = 3$.

References

[1] H. Machida and I.G. Rosenberg, Maximal centralizing monoids and their relation to minimal clones, *Proceedings 41st International Symposium on Multiple-Valued Logic*, IEEE, 2011, 153–159.