Inverse semigroups and Leavitt path algebras

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Leavitt path algebras are linear associative algebras associated with directed graphs. They are closely related to graph C^*-algebras. Their development is an outgrowth of the work of W.G. Leavitt who showed in the 1960's that non-commutative rings fail to have the invariant basis number property (in a very strong sense).

With each directed graph we introduce an inverse semigroup that we refer to as the *Leavitt inverse semigroup* of the graph. The Leavitt inverse semigroup of a directed graph is a natural subsemigroup of the multiplicative semigroup of the corresponding Leavitt path algebra. We show that two directed graphs that have isomorphic Leavitt inverse semigroups have isomorphic Leavitt path algebras. By contracting spanning trees of certain subgraphs of a directed graph Γ to a point, we obtain a new directed graph $\bar{\Gamma}$ with the property that the Leavitt inverse semigroups of Γ and $\bar{\Gamma}$ are strongly Morita equivalent and the Leavitt path algebras of Γ and $\bar{\Gamma}$ are Morita equivalent. We make use of this construction to give necessary and sufficient conditions for two graphs to have isomorphic Leavitt inverse semigroups. As a consequence, we study some structural properties of Leavitt inverse semigroups and Leavitt path algebras, and we show in particular that Leavitt path algebras are 0-retracts of certain matrix algebras.

[1]Joint work with David Milan (University of Texas-Tyler) and Zhengpan Wang (Southwest University, China)