

Infinitary pp-definability over the real numbers with convex relations

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On a finite structure, the polymorphism invariant relations are exactly the primitively positively definable relations. On infinite domain structures, these concepts are different. Infinitary primitively positively definable sets are a reasonable step in between which allows infinite intersections.

We look at the structure $(\mathbb{R}, +, \cdot, c \mid c \in \mathbb{R}, S)$ of the real numbers with addition, scalar multiplication, constants and an additional relation S representing a convex set. Closing this structure under infinitary primitively positively definable relations divides the convex sets into 6 equivalence classes.

While the proof to this theorem is done with complicated elementary geometry, it has consequences to model theory such as it gives a proof for the fact that there is no locally closed clone between the clone of affine combinations and the clone of convex combinations.