Wiegandt's theory of Möbius products is related to D-posets

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In a 1959 paper Richárd Wiegandt [2] defined the notion of Möbius product of functions on arbitrary locally finite posets. It generalizes the *Dirichlet convolution* (aka *Möbius product*) of two arithmetic functions $f, g : \mathbb{N} \to \mathbb{C}$ defined by

$$(f * g)(n) = \sum_{d|n} f(d)g(n/d),$$

where the sum is over the positive divisors d of n. In the general setting we have a locally finite partially ordered set $(P, \leq)$, a fixed 2-variable function $\alpha : P \times P \to P$, and a commutative ring R, then the α-product of two functions $f, g : P \to R$ is defined by

$$(f \underset{\alpha}{\circ} g)(a) = \sum_{b \leq a} f(b)g(\alpha(a, b)).$$

Wiegandt determined when this product is commutative and associative. It turns out that his conditions are essentially equivalent to the definition of *D-posets* defined by František Kôpka and Ferdinand Chovanec [1] in 1994. We characterize those posets of height 3 that can be equipped with a commutative and associative Möbius product. It has nice connections with combinatorics, group theory and universal algebra.

References

[1] František Kôpka and Ferdinand Chovanec, D-posets, *Math. Slovaca* 44 (1994), 21–34.

[2] Richárd Wiegandt, On the general theory of Möbius inversion formula and Möbius product, *Acta Sci. Math.* 20 (1959), 64–80.