The semigroup of increasing functions on the rationals and its unique Polish topology

Michael Pinsker[1]
Technische Universität Wien

How much information about a topology on a set can be *reconstructed* from the knowledge that it is compatible with a given algebraic structure on the same set in the sense that the algebraic operations are continuous? This problem has been studied from various angles and for many classes of algebraic structures over the years, using techniques from several areas of mathematics. Particular interest has been given to additional requirements on the topology, for instance that the topology be Polish, i.e., induced by a complete metric and enjoying a countable dense subset.

As an example, one can show that the additive group of the real numbers allows several Polish topologies, whereas the same group considered as a vector space over the reals has a unique Polish topology, namely the standard Euklidean topology. In fact, it turns out that several prominent algebraic structures are so rigid that they have a unique compatible Polish topology.

We consider the semigroup of all increasing functions on the rational numbers, equipped with composition. One compatible Polish topology is the topology of pointwise convergence in which a sequence of functions converges if it is eventually constant for every argument. We develop new techniques to show that this topology is unique.

[1]Joint work with Clemens Schindler