On Automorphisms of Strong Semilattice of Groups

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Let Ω be a semilattice and $\{S_\alpha, \alpha \in \Omega\}$ be the family of semigroups. A semigroup S is said to be the semilattice of semigroups S_α, $\alpha \in \Omega$, if $S = \bigcup_{\alpha\in\Omega} S_\alpha$ where $S_\alpha \cap S_\beta = \emptyset$ for $\alpha \neq \beta$ and $S_\alpha S_\beta \subseteq S_{\alpha\beta}$. If for each pair $\beta \leq \alpha$ in Ω, $\phi_{\alpha,\beta}\colon S_\alpha \to S_\beta$ is a semigroup homomorphism such that:

(i) $\phi_{\alpha,\alpha} = \mathrm{id}_{S_\alpha}$ for any $\alpha \in \Omega$.

(ii) The homomorphisms are transitive: For any $\alpha, \beta, \gamma \in \Omega$ with $\gamma \leq \beta \leq \alpha$ $\phi_{\beta,\gamma} \circ \phi_{\alpha,\beta} = \phi_{\alpha,\gamma}$.

On $S = \bigcup_{\alpha\in\Omega} S_\alpha$ define a multiplication $*$ where for $s \in S_\alpha$ and $t \in S_\beta$,

$$s * t = \phi_{\alpha,\alpha\beta}(s)\phi_{\beta,\alpha\beta}(t).$$

Then S forms a semigroup, denoted $S = [\Omega; S_\alpha, \phi_{\alpha,\beta}]$, known as a *strong semilattice of semigroups*. The homomorphisms $\phi_{\alpha,\beta}$ are called *structure homomorphisms*, Ω is called the *structure semilattice of* S, and the semigroups S_α are called the *components of* S. If each component S_α of S is a group, then S forms a semigroup known as *strong semilattice of groups*.

In this talk we shall discuss automorphisms of the strong semilattice of groups. We relate them to the isomorphisms and automorphisms of underlying groups. We first provide some results where the underlying semilattice has trivial automorphism group. We provide constructions for automorphisms of string semilattice of groups from structure homomorphisms and component isomorphisms. Finally we provide a construction for non-trivial automorphisms of semilattices. We also illustrate our results with some particular examples.

References

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