

Fraser-Horn-Hu Property for Ordered Algebras

Boža Tasić

Toronto Metropolitan University

Fraser and Horn ([1]), and independently Hu ([2]), studied varieties \mathcal{V} of algebras satisfying the property that for every $\mathbf{A}_1, \mathbf{A}_2 \in \mathcal{V}$, every congruence $\alpha \in \text{Con}(\mathbf{A}_1 \times \mathbf{A}_2)$ is a product congruence, i.e., $\alpha = \alpha_1 \times \alpha_2$ for some $\alpha_i \in \text{Con} \mathbf{A}_i$, $i = 1, 2$. Varieties of rings with identity and congruence distributive varieties of algebras satisfy this property. It is easy to show that an algebra $\mathbf{A}_1 \times \mathbf{A}_2$ has the Fraser-Horn-Hu property if and only if the map $(\alpha_1, \alpha_2) \mapsto \alpha_1 \times \alpha_2$ is a lattice isomorphism from $\text{Con} \mathbf{A}_1 \times \text{Con} \mathbf{A}_2$ to $\text{Con}(\mathbf{A}_1 \times \mathbf{A}_2)$. The property was later referred to in the literature as the Fraser-Horn-Hu property. It turns out that the Fraser-Horn-Hu property is a Mal'cev condition for varieties. In this paper we generalize this property to varieties of ordered algebras. The classic result of Fraser, Horn and Hu follows as a special case.

References

- [1] G. A. Fraser, A. Horn, Congruence relations in direct products. Proc. Amer. Math. Soc. **26**, 390-394 (1970) Partially ordered algebraic structures. (Pergamon Press, New York, 1963)
- [2] T. K. Hu, On equational classes of algebras in which congruences on finite products are induced by congruences on their factors. Manuscript (1970)