Representation of lattices by weak congruence lattices, in general, and a case of groups

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All weak congruences of an algebra make an algebraic lattice, with the diagonal element being a special element in this lattice. Recently we have been working on the characterisation of various classes of groups by their weak congruence lattices. We have defined lattices with normal elements with axioms that capture main properties of weak congruence lattice of groups. In this way, we characterised well-known classes of groups as Hamiltonian, Dedekind, abelian, solvable, nilpotent and many others in purely lattice theoretic terms. Lattices with normal elements are defined with several technical postulates; however, the open problem is to find a complete list of axioms for characterisation of lattices of weak congruences of groups. Another more general old (still open problem) of similar kind is the problem of representation of algebraic lattices by weak-congruence lattice. Again, there are a lot of necessary conditions that should be satisfied in order an algebraic lattice (with fixed element for diagonal) to be a lattice of weak congruences of an algebra. In our talk, several aspects of the mentioned problems will be elaborated.

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