Decidability Problems in the Plactic Monoid

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The plactic monoid is a monoid first studied in depth by Lascoux and Schützenberger [1, 2]. It endows the set of Semistandard Young tableaux with a natural monoid structure, and became an algebraic structure of interest when Lascoux and Schützenberger used it to prove the Littlewood-Richardson rule for Schur functions. It is naturally applicable to studying algebraic combinatorics and representation theory due to its link to Young tableaux, but has also been applied to algebraic geometry [3] and Lie theory via crystal basis theory [4].

When a monoid is finitely presented, a natural decision problem is whether this monoid has decidable first order theory. In this talk I will present the definition of a finitely presented plactic monoid, and prove that all such monoids have decidable first order theory.

References

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