Associative spectra of binary operations

Tamás Waldhauser[1]
University of Szeged

For a groupoid $\mathbb{A} = (A; \circ)$, let $s_n(\mathbb{A})$ denote the number of term operations induced by bracketings of the "product" $x_1 \circ \cdots \circ x_n$. The sequence $s_n(\mathbb{A})$ $(n = 1, 2, 3, \dots)$, called the *associative spectrum* of $\mathbb{A}$, measures how far the operation $\circ$ is from being associative: the larger the spectrum, the less associative the operation. The extremal cases are $s_n(\mathbb{A}) = 1$ (when $\circ$ is associative) and $s_n(\mathbb{A}) = \frac{1}{n}\binom{2n-2}{n-1}$ (we can say in this case that $\circ$ is antiassociative). Between these two extrema, we find a whole spectrum(!) of associative spectra.

The associative spectrum was introduced by Béla Csákany [3]; since then, several authors investigated related concepts [2, 8, 1, 5, 4], often rediscovering earlier results. In the talk I will first give an overview of these works, and then I will focus on a systematic study of associative spectra of graph algebras [6, 7] and on some recent results about quasigroups.

References

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[1] The most recent results presented in the talk constitute a joint work with Erkko Lehtonen.