The least left n-trinilpotent dimonoid congruence on the free trioid

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Trioids and free monogenic trioids first appeared in [1]. A *trioid* is a nonempty set T equipped with three binary associative operations $\dashv$, $\vdash$, and $\perp$ satisfying the following axioms:

$$(x \dashv y) \dashv z = x \dashv (y \vdash z), \quad (x \vdash y) \dashv z = x \vdash (y \dashv z),$$

$$(x \dashv y) \vdash z = x \vdash (y \vdash z), \quad (x \dashv y) \dashv z = x \dashv (y \perp z),$$

$$(x \perp y) \dashv z = x \perp (y \dashv z), \quad (x \dashv y) \perp z = x \perp (y \vdash z),$$

$$(x \vdash y) \perp z = x \vdash (y \perp z), \quad (x \perp y) \vdash z = x \vdash (y \vdash z)$$

for all $x, y, z \in T$. Recall the construction of the free trioid of an arbitrary rank.

As usual, $\mathbb{N}$ denotes the set of all positive integers. Let X be an arbitrary nonempty set, and let $F[X]$ be the free semigroup on X. For every word ω over X the length of ω is denoted by ℓ_ω. For any $n, k \in \mathbb{N}$ and $L \subseteq \{1, 2, \ldots, n\}, L \neq \varnothing$, we let $L + k = \{m + k \,|\, m \in L\}$. Define operations $\dashv$, $\vdash$, and $\perp$ on the set

$$F = \{(w, L) \,|\, w \in F[X],\, L \subseteq \{1, 2, \ldots, \ell_w\},\, L \neq \varnothing\}$$

by

$$(w, L) \dashv (u, R) = (wu, L), \quad (w, L) \vdash (u, R) = (wu, R + \ell_w),$$

$$(w, L) \perp (u, R) = (wu, L \cup (R + \ell_w))$$

for all $(w, L), (u, R) \in F$. According to [3], $(F, \dashv, \vdash, \perp)$ is the free trioid.

If ρ is a congruence on a trioid $(T, \dashv, \vdash, \perp)$ such that $(T, \dashv, \vdash, \perp)/\rho$ is a left (right) n-trinilpotent trioid, we say that ρ is a *left (right) n-trinilpotent congruence* [4]. If ρ is a congruence on a trioid $(T, \dashv, \vdash, \perp)$ such that two operations of $(T, \dashv, \vdash, \perp)/\rho$ coincide and $(T, \dashv, \vdash, \perp)/\rho$ is a dimonoid, we say that ρ is a *dimonoid congruence* [2]. A dimonoid congruence ρ on a trioid $(T, \dashv, \vdash, \perp)$ is called a $d^{\perp}_{\dashv}$*-congruence* [2] if the operations $\dashv$ and $\perp$ of $(T, \dashv, \vdash, \perp)/\rho$ coincide. If ρ is a congruence on a trioid $(T, \dashv, \vdash, \perp)$ such that the operations of $(T, \dashv, \vdash, \perp)/\rho$ coincide and $(T, \dashv, \vdash, \perp)/\rho$ is a left (right) n-nilpotent semigroup, we say that ρ is a *left (right) n-nilpotent semigroup congruence.*

We characterize the least left (right) n-trinilpotent $d^{\perp}_{\dashv}$-congruence and the least left (right) n-nilpotent semigroup congruence on the free trioid.

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References

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