On symmetric term operations in finite Taylor algebras

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In [1] it was proved that any finite Taylor algebra has a WNU term operation, which is an operation satisfying the following identity:

$$w(x, y, \dots, y) = w(y, x, y, \dots, y) = \dots = w(y, y, \dots, y, x).$$

As a generalization of this result, someone could ask for an arbitrary tuple of variables $(x, x, y, y, y, z, z, z, z, z, z, z, z)$ whether a finite algebra has a term operation satisfying all the identities coming from the permutation of this tuple.

We significantly generalized the original claim by proving that the existence of a WNU term operation of an odd arity n implies the existence of an n-ary term operation that is symmetric on all tuples with x and y. Equivalently, this means that the operation is symmetric on all the tuples containing just 2 different elements.

Additionally, we show that this result cannot be generalized for tuples with 3 variables x, y and z. That is, there exists a finite algebra having a WNU term operation of any odd arity and not satisfying any of such identities with at least three variables.

A stronger claim can be proved for the bounded width case, that is, when the algebra has WNUs of all arities $n \geq 3$. Such algebras have a term operation of any arity n that is symmetric on all good tuples, where a tuple t is good if there do not exist disjoint sets S_1 and S_2 such that t contains an equal number of elements from S_1 and S_2. For instance, the tuple (x, y, y, z, z, z) is not good but (x, y, y, z, z, z, z) is good. This result cannot be generalized further and there exists an algebra with bounded width not satisfying any identities coming from a bad tuple of variables.

In the talk we discuss these results and their applications for the complexity of the (Promise) Constraint Satisfaction Problem.

References

[1] M. Maróti, R. McKenzie, Existence theorems for weakly symmetric operations, *Algebra Universalis* 59, 2008, no. 3-4, 463–489.

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