Orthomodular bands

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An orthomodular band is an idempotent semigroup with zero $(A, \cdot, 0)$, where (i) every initial segment under the natural ordering of A is an orthomodular lattice with $\cdot$ as its meet operation (so, the band is necessarily normal), (ii) the corresponding sectional orthocomplementations are compatible in a certain sense. Motivating examples come, e.g., from Rickart rings and Baer semigroups, and include also the meet reduct of every orthomodular or generalized orthomodular lattice. We show that orthomodular bands can be equivalently defined replacing the family of sectional orthocomplementations with an appropriate binary operation considered as subtraction. Moreover, if A is right (left) normal, then its multiplication $\cdot$ also can be defined in terms of this operation.