The semigroup $U_n(L)$ of upper triangular matrices over idempotent semifields

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In this talk we discuss the semigroup of $n \times n$ upper triangular matrices $U_n(L)$ over an idempotent semifield arising from a linearly ordered abelian group L. The motivating example is that of the tropical semiring $\mathbb{T}$ arising from the additive group $\mathbb{R}$ of real numbers. It is known that the semigroup $U_n(L)$ is not regular for $n \geq 3$. Green's relations $\mathcal{R}$ and $\mathcal{L}$ coincide with their *-analogues $\mathcal{R}^*$ and $\mathcal{L}^*$, so that $U_n(L)$ is also not abundant for $n \geq 3$. We therefore consider the extensions $\widetilde{\mathcal{R}}, \widetilde{\mathcal{L}}, \widetilde{\mathcal{H}}$ and $\widetilde{\mathcal{D}}$ of Green's relations $\mathcal{R}, \mathcal{L}, \mathcal{H}$ and $\mathcal{D}$ on $U_n(L)$. We first show that $U_n(L)$ is *Fountain*, in fact there are unique idempotents of $U_n(L)$ in every $\widetilde{\mathcal{R}}$-class and $\widetilde{\mathcal{L}}$-class. The idempotent generated subsemigroup of $U_n(L)$ consists of all unitriangular matrices (that is, those with only the group identity down the leading diagonal). We show that $U_n(L)$ is an aperiodic semigroup and consider some of the identities it satisfies.

This is joint work with Marianne Johnson and Victoria Gould.

References

[1] J. S. Golan, *Semirings and their applications*, Kluwer Academic Publishers, 2010.

[2] S. N. Il'in, Regularity Criterion for Complete Matrix Semirings, *Mathematical Notes* 70, 2001, 329–336. (Translated from Matematicheskie Zametki.)

[3] M. Johnson and M. Kambites, Convexity of tropical polytopes, *Linear Algebra and its Applications* 485, 531–544, 2015.

[4] K. H. Kim, *Boolean matrix theory and applications*, Monographs and Textbooks in Pure and Applied Mathematics, Marcel Dekker, New York 1982.

[5] S. Margolis and B. Steinberg, Projective indecomposable modules for monoid algebras, `arXiv:1706.05425`.

[6] M. Taylor, *On upper triangular tropical matrix semigroups, tropical matrix identities and $\mathbb{T}$-modules*, PhD Thesis, Manchester, 2016.

[7] D. Wilding, M. Johnson, M. Kambites, Exact rings and semirings, *J. Algebra* 388, 2013, 324–337.

[8] D. Wilding, *Linear Algebra over Semirings*, PhD Thesis, Manchester, 2014.