Near-rings are very useful

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Near-rings are "rings with just one distributive law and with possibly non-commutative addition". Standard examples are collections $M(G)$ of all mappings from a group $(G, +)$ into itself, with point-wise addition and composition of mappings. If $(G, +)$ is abelian and if one only takes endomorphisms ("linear maps"), one gets rings; so near-rings can be viewed as the "non-linear generalizations of rings".

Kalle Kaarli has contributed many deep results to near-ring theory; this is now a sophisticated theory with many applications. In this talk, I want to present an especially useful class of near-rings N, the "planar" ones. They are characterized by the property that all equations $xa = xb + c$ have a unique solution x, unless $xa = xb$ holds for all $a, b \in N$. If $N^* = N \setminus \{0\}$ and one takes the collection $\boldsymbol{B}$ of all subsets of N of the form $aN^* + b$ (with $a \neq 0$) and their translates, one gets balanced incomplete block designs. It will be described why they are extremely useful for the design of statistical experiments, especially in biology and medicine.