

The sequent calculus of skew monoidal categories

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Szlachányi’s skew monoidal categories [1] are a well-motivated variation of monoidal categories in which the unitors and associator are not required to be natural isomorphisms, but merely natural transformations.

We present a sequent calculus for skew monoidal categories, building on the recent formulation by Zeilberger of a sequent calculus for the Tamari order [2]. In this calculus, antecedents consist of a stoup (an optional formula) followed by a context (a list of formulae), and the connectives tensor and unit behave like in non-commutative multiplicative linear logic (the logic of monoidal categories) except that the left rules may only be applied in stoup position. We show the admissibility of two forms of the cut rule (stoup cut and context cut) and prove the calculus sound and complete with respect to the free skew monoidal category. We introduce an equivalence relation on sequent calculus derivations and prove that there is a one-to-one correspondence between equivalence classes of derivations and maps in the free skew monoidal category. Finally, we identify a subcalculus of focused derivations, and establish that it contains exactly one canonical representative from each equivalence class. This gives us algorithms both for deciding equality of maps in the free skew monoidal category and for enumerating any homset without duplicates, in particular, for deciding whether there is a map. We have formalized this development in the dependently typed programming language Agda.

This is joint work with Niccolò Veltri and Noam Zeilberger.

References

- [1] K. Szlachányi, Skew-monoidal categories and bialgebroids, *Adv. Math.* 231(3–4), 2012, 1694–1730.
- [2] N. Zeilberger, A sequent calculus for a semi-associative law, in D. Miller (ed.), *FSCD 2017*, v. 84 of *Leibniz Int. Proc. in Inform.*, Dagstuhl Publishing, 2017, article 33.