# Solution sets of systems of equations

Tamás Waldhauser
University of Szeged
Szeged, Hungary

For any field $K$, the set of all solutions of a system of homogeneous linear equations in $n$ variables over $K$ is a subspace of $K^n$; conversely, every subspace arises as the solution set of a system of homogeneous linear equations. In other words, solution sets can be characterized as subsets of $K^n$ that are closed under linear combinations.

We would like to generalize this basic result by characterizing solution sets of systems of equations over general algebras. It is straightforward to verify that if $T \subseteq A^n$ is the set of all solutions of a system of equations over an algebra $\mathbf{A} = (A; F)$, then $T$ is closed under the centralizer clone $F^*$ consisting of those operations on $A$ that commute with every element of $F$. This motivates us to study algebras $\mathbf{A} = (A; F)$ for which the converse is also true, i.e., solution sets can be characterized as sets being closed under $F^*$. In this case we say that $\mathbf{A}$ has property (SCUC), where SCUC stands for "Solution set $\equiv$ Closed Under Centralizer".

Our main results are the following:

- Every two-element algebra has property (SCUC).
- A finite lattice has property (SCUC) if and only if it is a Boolean lattice.
- A finite semilattice has property (SCUC) if and only if it is (the semilattice reduct of) a distributive lattice.

This is a joint work with Endre Tóth (University of Szeged).