

# Corrigendum to “ On homological classification of pomonoids by regular weak injectivity properties of $S$ -posets”

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In Theorem 5.2 of our paper [1] only conditions (i) and (ii) (and not (iii)) are equivalent. Thus the theorem and its proof should be following.

**Theorem 5.2.** *The following conditions are equivalent for a pomonoid  $S$ :*

- (i) *all regularly divisible right  $S$ -posets are regularly principally weakly injective,*
- (ii) *for every element  $s \in S$  there exist  $r, r_1, \dots, r_n, s_1, \dots, s_n, s'_1, \dots, s'_n \in S$  and left po-cancellable elements  $c_1, \dots, c_n \in S$  such that*

$$\begin{aligned}
 c_1 s_1 &\leq r_1 s \leq c_1 s'_1 \\
 c_2 s_2 &\leq r_2 s_1 \leq r_2 s'_1 \leq c_2 s'_2 \\
 c_3 s_3 &\leq r_3 s_2 \leq r_3 s'_2 \leq c_3 s'_3 \\
 &\dots \\
 c_n s_n &\leq r_n s_{n-1} \leq r_n s'_{n-1} \leq c_n s'_n \\
 s &= s s_n = s s'_n.
 \end{aligned} \tag{4}$$

**Proof.** (i)  $\Rightarrow$  (ii) follows as in [1].

(ii)  $\Rightarrow$  (i). Assume (ii) holds. Let  $A_S$  be a regularly divisible right  $S$ -poset,  $s \in S$ , and  $f : sS \rightarrow A$  an  $S$ -poset morphism. Then for  $s$  we have inequalities and equalities as in (4). Hence  $f(s) = f(s)s_n = f(s)s'_n$ . Using regular divisibility of  $A$ , there exists  $a_1 \in A$  such that  $f(s) = a_1 c_n$ . Consequently,

$$f(s) = a_1 c_n s_n \leq a_1 r_n s_{n-1} \leq a_1 r_n s'_{n-1} \leq a_1 c_n s'_n = f(s),$$

and so  $f(s) = a_1 r_n s_{n-1} = a_1 r_n s'_{n-1}$ . Again, by the regular divisibility of  $A$ ,  $a_1 r_n = a_2 c_{n-1}$  for some  $a_2 \in A$ . Thus

$$\begin{aligned}
 f(s) &= a_2 c_{n-1} s_{n-1} \leq a_2 r_{n-1} s_{n-2} \leq a_2 r_{n-1} s'_{n-2} \leq a_2 c_{n-1} s'_{n-1} \\
 &= a_1 r_n s'_{n-1} = f(s)
 \end{aligned}$$

and  $f(s) = a_2 r_{n-1} s_{n-2} = a_2 r_{n-1} s'_{n-2}$ . In this way we finally arrive at  $f(s) = a_n r_1 s$  for some  $a_n \in A$ , i.e.  $f = \lambda_{a_n r_1}$ . So  $A$  is regularly principally weakly injective by Proposition 3.3.  $\square$

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## References

- [1] X. Zhang, V. Laan, *On homological classification of pomonoids by regular weak injectivity properties of  $S$ -posets*, Cent. Eur. J. Math. **5**(1) (2007), 181–200.