Seminar "Algebra ja tema rakendused" Vana-Otepääl
Workshop "Algebra and its applications" at Vana-Otepää
16.–18. 05. 2003

Abstracts

On supported algebras

Janis Cirulis
University of Latvia

Given a (countable) set of variables Var, we call a supported algebra a pair (A, spp), where A is an algebra and spp is a relation between subsets of Var and elements of A such that

- every set $(X : X \text{ spp } a)$ is a filter on Var, and
- every set $(a : X \text{ spp } a)$ is a subalgebra of A.

Here, elements of A are thought of as depending on variables. Where X spp a, we say that X is a support of a; in this case, a does not depend on variables outside of X. We characterize those supported algebras which can be represented by certain algebras of functions. The problem comes from algebraic logic.

Endoprimal Abelian groups of rank 1

Kalle Kaarli
University of Tartu

An algebra is called endoprimal if only its term functions permute with all of its endomorphisms. In [1] we solved, in particular, the endoprimality problem for the class of abelian groups that split into a direct sum of a torsionfree group of rank 1 and a torsion group. Here we generalise that result to arbitrary abelian groups of torsionfree rank 1, that is, to such groups whose quotient by the torsion part T is of rank 1. Given a mixed abelian group A, its nucleus N is a subring of the field of rational numbers $\mathbb{Q}$ generated by the inverses of prime numbers p such that the p-component T_p of A is zero and the quotient A/T is p-divisible. Clearly the group A has a natural structure of an N-module. When studying endoprimality of abelian groups, it is conveniemt to consider them as modules over their nucleus and actually to study the endoprimality of those modules. Our main result is the following.

Theorem. Let A be an abelian group of torsionfree rank 1 with nucleus N, torsion part T and p-components T_p. Denote by P the set of primes p such that A/T is p-divisible. The group A is endoprimal if and only if T is unbounded and for every p in P one of the following three conditions is satisfied: 1) $T_p = 0$, 2) T_p is not reduced, 3) T_p is not a direct summand of A.

[1] Kaarli, K. and Márki, L., *Endoprimal abelian groups.* J. Austral. Math. Soc. (Series A) **67** (1999), 412-428.

On equalizer flat acts

Mati Kilp
University of Tartu

In Comm. Algebra 30(3) (2002), 1475 - 1498, Bulman-Fleming and Kilp developed various notions of flatness of a right act A_S over a monoid S that are based on the extent to which the functor $A_S \otimes -$ preserves equalizers. In Semigroup Forum 65 (3) (2002), 428–449, Bulman-Fleming discussed in detail one of these notions, annihilator-flatness. The present paper is devoted to two more of these notions, weak equalizer-flatness and strong torsion-freeness. An act A_S is called weakly equalizer-flat if the functor $A_S \otimes -$ 'almost' preserves equalizers of any two homomorphisms into the left act $_SS$, and strongly torsion-free if this functor 'almost' preserves equalizers of any two homomorphisms from $_SS$ into the Rees factor act $_S(S/Sc)$, where c is any right cancellable element of S. (The adverb 'almost' signifies that the canonical morphism provided by the universal property of equalizers may be only a monomorphism rather than an isomorphism.) From the definitions it is clear that flatness implies weak equalizer-flatness, which in turn implies annihilator-flatness, and it was known already that both of these implications are strict. A monoid is called right absolutely weakly equalizer-flat if all of its right acts are weakly equalizer-flat. In this paper we prove a result which implies that right PP monoids with central idempotents are absolutely weakly equalizer-flat. We also show that for a relatively large class of commutative monoids, right absolute equalizer-flatness and right absolute annihilator-flatness coincide. Finally, we provide examples showing that the implication between strong torsion-freeness and torsion-freeness is strict.

Affine completion of some Ockham algebras

Vladimir Kutšmei
University of Tartu

For any algebra **A** of the variety generated by the class of all Stone and Kleene algebras we construct its extension **B** with the property that every local polynomial function of **A** is the restriction of a suitable polynomial of **B**. Moreover, we characterize local polynomial functions of **A** as compositions of polynomials and special unary functions.

On wreath product of SET-valued functors

Valdis Laan
University of Tartu

We study when the functor of tensor multiplication by the wreath product of functors G and H preserves limits of certain type in terms of similar properties for G and H.

On loops and their multiplication groups

Markku Niemenmaa
University of Oulu

We say that a groupoid Q is a *loop* if Q has a unique division and a neutral element (thus loops are nonassociative versions of groups). The mappings $L_a(x) = ax$ and $R_a(x) = xa$ define two permutations on Q for every $a \in Q$ and the permutation group $M(Q) = \langle L_a, R_a : a \in Q \rangle$ is called the multiplication group of Q. By $I(Q)$ we denote the stabilizer of the neutral element and $I(Q)$ is called the inner mapping group of Q.

Many properties of loops can be reduced to the properties of connected transversals in the multiplication group. The precise definition is as follows: If G is a group, $H \leq G$ and A and B are two left transversals to H in G such that the commutator subgroup $[A, B]$ is contained in H then we say that A and B are H-connected in G. By H_G we denote the core of H in G (the largest normal subgroup of G contained in H). If Q is a loop then it is rather easy to see that the core of $I(Q)$ in $M(Q)$ is trivial and $A = \{L_a : a \in Q\}$ and $B = \{R_a : a \in Q\}$ are $I(Q)$-connected transversals in $M(Q)$.

In my talk I shall explore the relation between loops, their multiplication groups and connected transversals.

Congruence compactness for algebras

Peeter Normak
Tallinn Pedagogical University

We discuss the problem how the property "congruence compactness" of an algebra is related to the congruences on this algebra and to the semilattice structure of its CC-congruences (A congruence ρ on an algebra A is called a CC-congruence if the factor algebra A/ρ is congruence compact). Some properties of the (upper) semilattice of CC-congruences of an arbitrary algebra are discussed. The concept of congruence compactness is generalized to subclasses of algebras and congruences. General concepts and results are illustrated by monoids and acts.

Operads, cohomology, and deformations

Eugen Paal
Tallinn Technical University

Basic notions and ideas of the operad theory are presented on the ground level. Connections with the cohomology theory and deformation theory of algebras are explained. Examples are provided.

Groups of order < 32 and their endomorphism semigroups

Peeter Puusemp
Tallinn Technical University

The full characterization of groups of order < 32 by their endomorphism semigroups in the class of all groups is given.

Syntactic monoids of top-down deterministic tree languages

Magnus Steinby
University of Turku

As noted already by Magidor and Moran [9], the family DRec of *dT tree languages* recognized by *deterministic top-down (dT) tree recognizers* (also called *deterministic root-to-frontier (DR) tree recognizers*) forms a proper subfamily of the family Rec of all regular tree languages. Although even some very simple tree languages are excluded, this family has many interesting properties and a rather extensive literature (cf. [4, 5, 7, 8] also for further references). In particular, any context-free language is the yield of a dT tree language. On the other hand, DRec does not form a variety of tree languages [11, 12], and hence it cannot be characterized in the usual way by syntactic algebras [11, 12] or by syntactic monoids [13, 10].

Any dT tree language T is determined by its *path languages*; for each leaf symbol x there is a path language T_x formed by words describing the paths from the root to a leaf labeled with x in some tree belonging to T. In fact, T is *closed* in the sense that it contains every tree that can be assembled from paths belonging to trees in T, and this property characterizes dT tree languages [1, 14].

In this lecture we first present a Nerode-like theorem for dT tree languages, where the Nerode congruence of a tree language T is derived from the path languages of T. The *Nerode path congruence* of a tree language T corresponds in a natural way to the *minimal dT recognizer* of T [3]. Then, extending the same idea, we introduce *syntactic path congruences* and *syntactic path monoids* of tree languages. The syntactic path monoid $PM(T)$ of a tree language T is finite iff T is in DRec. Moreover, $PM(T)$ can be computed as the transition monoid of an automaton associated with T. The lecture is mainly based on joint work with F. Gécseg [6].

[1] Courcelle, B.: A representation of trees by languages I. *Theoretical Computer Science* **6** (1978), 255-279.

[2] Gécseg, F. and Peák, I.: *Algebraic theory of automata.* Akadémiai Kiadó, Budapest 1972.

[3] Gécseg, F. and Steinby, M.: Minimal ascending tree automata. *Acta Cybernetica* **4** (1978), 37-44.

[4] Gécseg, F. and Steinby, M.: *Tree automata.* Akadémiai Kiadó, Budapest 1984.

[5] Gécseg, F. and Steinby, M.: Tree languages. *Handbook of Formal Languages*, Vol. 3 (eds. G. Rozenberg and A. Salomaa), Springer-Verlag, Berlin 1997, 1-69.

[6] Gécseg and Steinby, M.: Minimal recognizers and syntactic monoids of DR tree languages. *Words, Semigroups and Transductions* (eds. M. Ito, G. Paun and S. Yu), World Scientific, Singapore 2001, 155-167.

[7] Jurvanen, E.: The Boolean closure of DR-recognizable tree languages. *Acta Cybernetica* **10** (1992), 255-272.

[8] Jurvanen, E.: *On tree languages defined by deterministic root-to-fronties recognizers.* Doctoral Thesis, Department of Mathematics, University of Turku, Turku 1995.

[9] Magidor, M. and Moran, G.: *Finite automata over finite trees.* Technical Report **30**, Hebrew University, Jerusalem 1969.

[10] Nivat, M. and Podelski, A.: Minimal ascending and descending tree automata. *SIAM J. Computation* **26** (1997), 39-58.

[11] Steinby, M.: Syntactic algebras and varieties of recognizable sets. *Les Arbres en Algèbre et en Programmation* (Proc. 4th CAAP, Lille 1979), University of Lille, Lille 1979, 226-240.

[12] Steinby, M.: A theory of tree language varieties. *Tree Automata and Languages* (eds. M. Nivat and A. Podelski), North-Holland, Amsterdam 1992, 57-81.

[13] Thomas, W.: Logical aspects in the study of tree languages. In: *9th Colloquium on Trees in Algebra and Programming* (Proc. 9th CAAP, Bordeaux 1984), ed. B. Courcelle, Cambridge University Press, London 1984, 31-49.

[14] Virágh, J.: Deterministic ascending tree automata I. *Acta Cybernetica* **5** (1980), 33-42.

Uniqueness in injectivity properties of acts over monoids

Kati Tabur

University of Tartu

Let S be a monoid. A right S-act A_S is called injective if for every homomorphism $f : B_S \to A_S$ and every monomorphism $\iota : B_S \to C_S$ there exists a homomorphism $g : C_S \to A_S$ such that $g\iota = f$. If the condition is satisfied for B_S a right ideal of S (principal right ideal of S, S) then A_S is called weakly injective (principally weakly injective, divisible). We study properties arising by requiring that the homomorphism g is unique.