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Abstracts

On continuity of A-linear maps

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While I was working on generalization of Serre-Swan-Mallios Theorem which gives us a "bridge" between algebraic K-theory and topological K-theory, it became necessary to establish the continuity of all A-linear maps between finitely generated projective A-modules, where A is an (topological) algebra. This problem was reduced to the following problem: under which conditions on topological algebra A all A-linear maps $f : A^n \to A^m$ are continuous for every $n, m \in \mathbb{N}$? For unital algebra A the positive answer was known for long time already. In this talk we will introduce a class of (nonunital) topological algebras for which the answer is still positive.

Topological algebras

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A short survey about the theory of topological algebras is given. Several properties of Gelfand-Mazur algebras are described. Since every topological algebra is also a bornological algebra, then we can use bornological properties of topological algebra for the description of the structure of topological algebras.

Weak BCK*-algebras and orthosemilattices

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A dual BCK-algebra, or BCK*-algebra is a poset $(A, \leq)$ having the greatest element 1 and considered together with a binary operation $\to$ such that, for all $x, y, z \in A$,

(0) $x \leq y$ iff $x \to y = 1$,
(1) $x \leq (x \to y) \to y$,
(2) $x \to y \leq (y \to z) \to (x \to z)$.

A BCK*-algebra is commutative if, one more axiom

(3) $x \to (x \to y) = y \to (y \to x)$.

A weak [commutative] BCK*-algebra satisfies (0), (1), [(3)] and a weaker axiom

(2^-) if $x \leq y$, then $y \to z \leq x \to z$.

In the talk, commutative weak BCK*-algebras will be shown to posess many structural properties of commutative BCK*-algebras. Moreover, it will be shown that the class of commutative weak BCK*-algebras is definitionally equivalent to that of orthosemilattices [2], known also as semilattices with sectionally antitone involutions [1].

[1] I. Chajda, *Lattices and semilattices having an antitone involution in every upper interval*, **Comment. Math. Univ. Carolinae**, 44 (2003), pp. 577–585.

[2] I. Chajda, R. Halaš, *An implication in orthologic*, **arXiv:quant-ph/0210083**, v2 (25 Apr. 2003).

Topologies and Lattice Structures in Rough Set Theory

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The talk presents some results appearing in papers [1, 2, 3]. We study lattice structures of rough approximations and rough sets determined by indiscernibility relations which are not necessarily reflexive, symmetric or transitive.

Any map f between two complete lattices has an adjoint g, and the pair (f, g) is a Galois connection, if and only if f is a complete join-morphism. Further, it is known that the rough set upper approximation operator determined by an arbitrary binary relation R on a universe U is a complete join-morphism on the power set of U; therefore it induces a Galois connection such that the adjoint is the lower approximation operation determined by the inverse relation of R. We point out that the main properties of rough approximation operators follow from the common and well-known properties of Galois connections on Boolean lattices.

Also the fixed points of rough approximation operations are considered. For example, we show that the set of fixed points of the upper approximation operation determined by a reflexive indiscernibility relation R forms an Alexandrov Topology $\mathcal{T}_R$ and if the relation R is also symmetric, the topology $\mathcal{T}_R$ is closed under complementation. Further, if the underlying relation R is reflexive and transitive, then the Alexandrov topology $\mathcal{T}_R$ is such that the upper approximation operator itself serves as the smallest neighborhood operator.

We present some observations on the ordered set of rough sets determined by different types of indiscernibility relations. We show that for tolerances and transitive binary relations the set of rough sets is not necessarily even a semilattice. We also prove that the set of rough sets determined by a symmetric and transitive binary relation forms a complete Stone lattice.

[1] Järvinen, J.: On the structure of rough approximations. Fundamenta Informaticae **53** (2002) 135–153.

[2] Järvinen, J.: The ordered set of rough sets. In Tsumoto, S., Slowinski, R., Komorowski, H.J., Grzymala-Busse, J.W., eds.: Rough Sets and Current Trends in Computing, 4th International Conference, RSCTC 2004, Uppsala, Sweden, June 1–5, 2004, Proceedings. Volume 3066 of Lecture Notes in Computer Science. Springer-Verlag, Berlin, Heidelberg (2004) 49–58.

[3] Järvinen, J., Kondo, M., Kortelainen, J.: Modal-like operators in boolean algebras, Galois connections and fixed points. Submitted to Fundamenta Informaticae (2006).

Sublattices of direct products of two lattices

Kalle Kaarli
University of Tartu

Compatible functions on Kleene algebras

Vladimir Kuchmei
University of Tartu

We discuss the following problem. Given a Kleene algebra $\mathbf{A}$, find a nice generating set for the clone of all compatible functions.

Morita equivalence of partially ordered monoids

Valdis Laan
University of Tartu

Two partially ordered monoids (*pomonoids*) are called *Morita equivalent* if the categories of ordered acts over them (also called *S-posets*) are poequivalent. We find elementary necessary and sufficient conditions for two pomonoids to be Morita equivalent. It turns out that Morita equivalent pomonoids share a number of properties. For example the congruence (or ideal) lattices of two Morita equivalent pomonoids are isomorphic.

Subfunctions and Burle's clones

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We consider operations on a fixed base set A. Let $\mathcal{C}$ be a class of operations on A. We say that f is a $\mathcal{C}$-subfunction of g, if $f = g(h_1, \ldots, h_n)$ for some $h_1, \ldots, h_n \in \mathcal{C}$. The $\mathcal{C}$-subfunction relation is a preorder on the set of all operations on A if and only if $\mathcal{C}$ is a clone.

In this presentation, we focus on those subfunction relations which are defined by the clones containing all unary operations on a finite base set. As shown by Burle, there are $k+1$ such clones on a k-element base set A and they constitute a chain in the lattice of clones on A. We investigate certain order-theoretical properties of these subfunction relations.

On clones on squarefree expanded groups

Peter Mayr
University of Linz

Is there a finite set with more than countably many clones that contain a Mal'cev operation? We will talk about an instance of this problem for clones on expanded groups. In particular we show that there are only finitely many polynomially inequivalent expansions of groups whose orders are a product of at most 3 distinct primes. This extends previous results by E. Aichinger and P. Mayr that characterize the clone of polynomial functions on any expanded group whose order is a product of 2 distinct primes and on any expanded group whose order is squarefree and whose congruences are linearly ordered. Still we do not have a proof for the full conjecture of P. M. Idziak that each squarefree group has only finitely many polynomially inequivalent expansions.

Principal varieties of finite congruences

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Equations have been a fundamental tool in mathematics for centuries, especially in algebra. The classical example in universal algebra is Birkhoff's theorem that establishes the connection between equational theories and varieties of algebras. A related result of Eilenberg and Schtzenberger (1976) connects sequences of equations and *varieties of finite algebras* (VFA) which are often called also pseudovarieties. Let us note that in this presentation we are interested only in algebras having a given finite signature Σ.

The VFAs have later been characterized by certain families of finite congruences of term algebras called *varieties of finite congruences* (VFC). In a VFC $\Gamma = \{\Gamma(X)\}_X$, the index X ranges over all finite alphabets, called *leaf alphabets* and used as generating sets of term algebras, and for each X, $\Gamma(X)$ is a filter of finite congruences on the term algebra $T_\Sigma(X)$; a congruence is *finite* if it has a finite number of classes. Moreover, there is a certain connection between $\Gamma(X)$ and $\Gamma(Y)$ for any X and Y.

Our main topic is an interesting special subclass of VFCs, the *principal varieties of finite congruences* (pVFC). For each X, the filter $\Gamma(X)$ of a pVFC Γ is *principal*, i.e., generated by a single congruence. This special class was introduced already by Steinby (1992) along with some general observations concerning VFCs, but it was not investigated any further.

Studying pVFCs is motivated from the algebraic point of view by the fact that all VFCs are decomposable into unions of *finitely generated VFCs*, and all finitely generated VFCs are also principal. A natural first question is whether all pVFCs are finitely generated, and this turns out not to be the case. Therefore the finitely generated VFCs are the very basic building blocks of VFCs, but the principal VFCs form an interesting superclass.

The structure of a pVFC is in general slightly more complicated than the structure of a finitely generated VFC, but we still retain the finiteness of all congruence lattices, which is an important feature from the viewpoint of applications. In fact, applications in tree language and automata theory are the second reason why principal varieties are especially appealing. For each VFA, and hence also for each VFC, there is a unique indexed family of tree languages, a *variety of tree languages* (VTL), which corresponds to these classes bijectively. This three-way connection has proven useful when defining concrete classes of tree languages. For many concrete tree language classes like definite, locally testable or piecewise testable tree languages, the corresponding class of congruences has natural and relatively simple properties, and congruences can be very useful also for further characterizations of the tree language classes. Often the easiest way to define a VTL is to use principal varieties of finite congruences which may or may not be finitely generated.

In addition to the elementary concepts related to principal varieties of finite congruences we investigate closer the general structural relations between the filters of a pVFCs. We answer for example the following question: If we know finitely many of the filters constituting a principal variety of finite congruences, how much do we know about the rest of the variety? Our result determines an interval of pVFCs where our partially known variety must be included. Moreover, each variety in this interval could be a possible solution. This interval is actually strongly connected to certain sets of identities determined by the corresponding VFA, which is not surprising, since each filter of a pVFC is generated by a fully invariant congruence on a term algebra, and one of Birkhoff's original results was that fully invariant congruences have a natural interpretation as a deductively closed class of identities. One further consequence of these investigations is that if we know only finitely many filters of a given VFC, we cannot in general say even whether it is principal or not.

We present also a bijective correspondence between the principal varieties of finite congruences and locally finite varieties of algebras, and as a consequence the finitely generated VFCs and the finitely generated varieties of algebras will correspond to each other bijectively as well. This result provides an interesting link from tree language theory to a widely researched subject in universal algebra. As an application of the theory we demonstrate how to construct a concrete tree language variety using principal varieties.

At the end of the talk we will give some pointers to related topics which might have some interesting connections to the presented results. Some topics that are closely related to pVFCs include for example equational logic, locally finite varieties, and the structural theory of finite algebras.

Implicit operations on some categories of finite universal algebras

Alexander Pinus
Novosibirsk State Technical University

Generalizations of some concepts and results by Eilenberg and Schtzenberger on pseudovarieties of finite algebras and their implicit operations are considered.

Some recent results on endomorphisms of groups

Peeter Puusemp
Tallinn University of Technology

On interval decomposition lattices

Sándor Radeleczki
University of Miskolc
(Joint work with S. Földes, Math. Institute of the Tampere University of Technology)

Intervals in binary or n-ary relations or other discrete structures generalise the concept of interval in a linearly ordered set. They are defined abstractly as closed sets of a closure system on a set V, satisfying certain axioms. Decompositions are partitions of V whose blocks are intervals, and they form an algebraic semimodular lattice. The properties of this lattice are explored.

Algebraic structures corresponding to Steiner triple systems

Ellen Redi
Tallinn University

On flatness properties of S-posets

Nikita Salnikov
Tallinn University

Formal implications and closure operators

Stefan Schmidt
Dresden University of Technology

On some properties of comma categories

Sergejs Solovjovs
University of Latvia

Let $\mathbf{A} \xrightarrow{U} \mathbf{X}$ be an adjoint functor. We show the necessary and sufficient conditions for the comma category $id_{\mathbf{X}} \downarrow U$ to be algebraic (coalgebraic) and monadic as well as consider factorization structures on $id_{\mathbf{X}} \downarrow U$.

On weighted order colimits and enriched points

Lauri Tart
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We present the notion of a weighted colimit from enriched category theory in the special case of order-enriched categories (specifically categories of ordered acts). This is followed by a discussion of convenient conceptual reformulations of the definition of weighted colimit for ordered acts and some important special colimits. Finally, we use the preceding work to arrive at a proper enriched presentation of order-geometric morphisms.

Around the Stone space's construction for distributive lattices

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Following George Grtzer [General Lattice Theory, Second Edition, 2003], we formalize the construction of a Stone space of a distributive lattice and the corresponding representation theorem. We fulfil all the proofs only in terms of an algebraic lattice (i.e., the lattice of all ideals of the underlying lattice) and realize the dual construction for coalgebraic lattices. Introducing the notions of a prime element and a cobase, we show that the Stone-type construction yields an i-topological space for an arbitrary cobase. It is the case of a topological space iff the cobase consists precisely of all prime elements, although an arbitrary cobase does not necessarily contain all prime elements.