Weak BCK*-algebras and orthosemilattices

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A dual BCK-algebra, or BCK*-algebra is a poset $(A, \leq)$ having the greatest element 1 and considered together with a binary operation $\to$ such that, for all $x, y, z \in A$,

(0) $x \leq y$ iff $x \to y = 1$,
(1) $x \leq (x \to y) \to y$,
(2) $x \to y \leq (y \to z) \to (x \to z)$.

A BCK*-algebra is commutative if, one more axiom

(3) $x \to (x \to y) = y \to (y \to x)$.

A weak [commutative] BCK*-algebra satisfies (0), (1), [(3)] and a weaker axiom

(2^-) if $x \leq y$, then $y \to z \leq x \to z$.

In the talk, commutative weak BCK*-algebras will be shown to posess many structural properties of commutative BCK*-algebras. Moreover, it will be shown that the class of commutative weak BCK*-algebras is definitionally equivalent to that of orthosemilattices [2], known also as semilattices with sectionally antitone involutions [1].

References

[1] I. Chajda, *Lattices and semilattices having an antitone involution in every upper interval*, **Comment. Math. Univ. Carolinae**, 44 (2003), pp. 577–585.

[2] I. Chajda, R. Halaš, *An implication in orthologic*, **arXiv:quant-ph/0210083**, v2 (25 Apr. 2003).