Topologies and Lattice Structures in Rough Set Theory

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Abstract

The talk presents some results appearing in papers [1, 2, 3]. We study lattice structures of rough approximations and rough sets determined by indiscernibility relations which are not necessarily reflexive, symmetric or transitive.

Any map f between two complete lattices has an adjoint g, and the pair (f, g) is a Galois connection, if and only if f is a complete join-morphism. Further, it is known that the rough set upper approximation operator determined by an arbitrary binary relation R on a universe U is a complete join-morphism on the power set of U; therefore it induces a Galois connection such that the adjoint is the lower approximation operation determined by the inverse relation of R. We point out that the main properties of rough approximation operators follow from the common and well-known properties of Galois connections on Boolean lattices.

Also the fixed points of rough approximation operations are considered. For example, we show that the set of fixed points of the upper approximation operation determined by a reflexive indiscernibility relation R forms an Alexandrov Topology $\mathcal{T}_R$ and if the relation R is also symmetric, the topology $\mathcal{T}_R$ is closed under complementation. Further, if the underlying relation R is reflexive and transitive, then the Alexandrov topology $\mathcal{T}_R$ is such that the upper approximation operator itself serves as the smallest neighborhood operator.

We present some observations on the ordered set of rough sets determined by different types of indiscernibility relations. We show that for tolerances and transitive binary relations the set of rough sets is not necessarily even a semilattice. We also prove that the set of rough sets determined by a symmetric and transitive binary relation forms a complete Stone lattice.

References

[1] Järvinen, J.: On the structure of rough approximations. Fundamenta Informaticae **53** (2002) 135–153.

[2] Järvinen, J.: The ordered set of rough sets. In Tsumoto, S., Slowinski, R., Komorowski, H.J., Grzymala-Busse, J.W., eds.: Rough Sets and Current Trends in Computing, 4th International Conference, RSCTC 2004, Uppsala, Sweden, June 1–5, 2004, Proceedings. Volume 3066 of Lecture Notes in Computer Science. Springer-Verlag, Berlin, Heidelberg (2004) 49–58.

[3] Järvinen, J., Kondo, M., Kortelainen, J.: Modal-like operators in boolean algebras, Galois connections and fixed points. Submitted to Fundamenta Informaticae (2006).