Polynomial functions on a class of finite groups

Peeter Puusemp
University of Tartu
Tartu, Estonia

We describe unary polynomial functions on noncommutative groups G that are semidirect products $A \rtimes_\alpha B$, where $A \cong \mathbb{Z}_p^2$ and $B \cong \mathbb{Z}_q$ with p and q different prime numbers, and $\alpha : B \to \operatorname{Aut} A$ a group homomorphism.

Since α is a homomorphism, $\mathbb{Z}_q$ is cyclic and $\operatorname{Aut} A \cong \mathrm{GL}_2(\mathbb{Z}_p)$, the homomorphism α is determined by a matrix $\alpha(1) = M$. We consider separately two cases: one when the matrix M has no eigenvalues and the other when M does have eigenvalues. The first one is a special case of what was studied in E. Aichinger's paper [1]. The second one has not been studied earlier.

Our main results are the following.

(i) If the matrix M has no eigenvalues, then the group G has $p^{4q}q^2$ unary polynomial functions.

(ii) If

$$M = \begin{pmatrix} \lambda & 0 \\ 0 & \mu \end{pmatrix}$$

with $\lambda \neq \mu$ and $\lambda, \mu \neq 1$, then the group G has $p^{4q}q^2$ unary polynomial functions.

(iii) If

$$M = \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}$$

with $\lambda \neq 1$, then the group G has $p^{3q}q^2$ unary polynomial functions.

(iv) If

$$M = \begin{pmatrix} 1 & 0 \\ 0 & \mu \end{pmatrix}$$

with $\mu \neq 1$, then the group G has $p^{2q+2}q^2$ unary polynomial functions.

(v) The case

$$M = \begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix}$$

is not possible.

References

[1] E. Aichinger, The polynomial functions on certain semidirect products of groups, *Acta Sci. Math.* (Szeged), **68** (2002), 63-81.