Uniform majority algebras

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We call a congruence on an algebra k-uniform if all of its classes have size k. A uniform algebra is one where every congruence has equal-sized classes — groups are an example.

A majority function is a ternary operation m satisfying $m(x, y, y) = m(y, x, y) = m(y, y, x) = y$. For example, lattices have the majority term $m(x, y, z) = (x \wedge y) \vee (x \wedge z) \vee (y \wedge z)$.

In 2004, Kalle Kaarli proved that finite uniform lattices are congruence permutable, settling a conjecture of Grätzer, Quackenbush and Schmidt. Gábor Czédli has tried to generalise this result from lattices to arbitrary majority algebras. So far, he has proved that two 2-uniform congruences on finite majority algebras permute.

We show that a 2-uniform congruence on a finite majority algebra permutes with an arbitary uniform congruence, if their meet is the zero congruence. This slightly improves Czédli's result.